36) f) Cont.

$tg\ 33.75°$

$$tg\,67.5° = \frac{\sin 67.5°}{\cos 67.5°}$$

$$= \frac{\sin 67.5°}{\sqrt{1-\sin^2 67.5°}}$$

Mas do item (a) já obteve-se que

$$\sin 67.5° = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

Daí

$$tg\,67.5 = \frac{\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}}{\sqrt{1-\frac{\sqrt{2}+1}{2\sqrt{2}}}}$$

$$= \frac{\sqrt{\sqrt{2}+1}}{\sqrt{\sqrt{2}-1}}$$

$$tg\,67.5° = \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$$

Mas:

$$tg\,67.5° = tg\,2\times 33.75°$$

$$\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \frac{2\,tg\,33.75°}{1-tg^2 33.75°} \quad (*)$$

Seja $\alpha \equiv tg\,33.75°$

Então (*) fica:

$$\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \frac{2\alpha}{1-\alpha^2}$$

$$\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}\,(1-\alpha^2) = 2\alpha$$

$$\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} - \alpha^2\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} - 2\alpha = 0$$

$$\alpha^2\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} + 2\alpha - \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = 0$$

$$\alpha = \frac{-2 \pm \sqrt{4+4\cdot\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)}}{2\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}}$$

$$= \frac{-\cancel{2} \pm \cancel{2}\sqrt{1+\frac{\sqrt{2}+1}{\sqrt{2}-1}}}{\cancel{2}\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}}$$

$$= \frac{-1 \pm \sqrt{\frac{2\sqrt{2}}{\sqrt{2}-1}}}{\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}} > 0$$

$$\Rightarrow \boxed{tg\,33.75° = \frac{-1+\sqrt{\frac{2\sqrt{2}}{\sqrt{2}-1}}}{\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}}}$$