• $x = \frac{1+\sqrt{5}}{4}$

Aqui,

$$\cos 2\theta = \frac{1+\sqrt{5}}{4}$$

$$2\cos^2\theta - 1 = \frac{1+\sqrt{5}}{4}$$

$$2\cos^2\theta = 1 + \frac{1+\sqrt{5}}{4}$$

$$\cos^2\theta = \frac{5+\sqrt{5}}{8}$$

• $x = \frac{1-\sqrt{5}}{4}$

$$\cos 2\theta = \frac{1-\sqrt{5}}{4}$$

$$2\cos^2\theta - 1 = \frac{1-\sqrt{5}}{4}$$

$$2\cos^2\theta = 1 + \frac{1-\sqrt{5}}{4}$$

$$\cos^2\theta = \frac{5-\sqrt{5}}{8}$$

isto é há 2 possibilidades

$$\boxed{\cos^2\theta = \frac{5+\sqrt{5}}{8} \text{ ou } \frac{5-\sqrt{5}}{8}}$$

47.

$$\sin^2\left(\frac{\pi}{8}+\frac{\theta}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{\theta}{2}\right)$$

Inicialmente notemos que

$$\sin^2(a+b) - \sin^2(a-b) =$$

$$= (\sin a\cos b + \sin b\cos a)^2 - (\sin a\cos b - \sin b\cos a)^2$$

$$= \sin^2 a\cos^2 b + 2\sin a\cos a\sin b\cos b + \sin^2 b\cos^2 a - (\sin^2 a\cos^2 b - 2\sin a\cos a\sin b\cos b + \sin^2 b\cos^2 a)$$

$$= \cancel{\sin^2 a\cos^2 b} + 2\sin a\cos a\sin b\cos b + \cancel{\sin^2 b\cos^2 a} - \cancel{\sin^2 a\cos^2 b} + 2\sin a\cos a\sin b\cos b - \cancel{\sin^2 b\cos^2 a}$$

$$= 4\sin a\cos a\sin b\cos b$$

$$= \sin 2a\sin 2b$$

$\therefore$

$$\sin^2(a+b) - \sin^2(a-b) = \sin 2a\sin 2b$$

Se tomarmos

$$a = \frac{\pi}{8}, \quad b = \frac{\theta}{2}$$

teremos