

$$34) 4(\cos^6 x + \sin^6 x) = 1 + 3\cos^2 2x$$

left-hand side

$$\begin{aligned}
 4(\cos^6 x + \sin^6 x) &= \\
 &= 4(\cos^6 x + (\sin^2 x)^3) \\
 &= 4(\cos^6 x + \underbrace{(1-\cos^2 x)^3}_{}) \\
 &= 4(\cos^6 x + (1-2\cos^2 x + \cos^4 x)(1-\cos^2 x)) \\
 &= 4(\cos^6 x + 1 - \cos^2 x - 2\cos^2 x + 2\cos^4 x + \cos^6 x \\
 &\quad - \cos^6 x) \\
 &= 4(1 - 3\cos^2 x + 3\cos^4 x) \\
 &= 4 - 12\cos^2 x + 12\cos^4 x //
 \end{aligned}$$

right-hand side

$$\begin{aligned}
 1 + 3\cos^2 2x &= 1 + 3(2\cos^2 x - 1)^2 \\
 &= 1 + 3(4\cos^4 x - 4\cos^2 x + 1) \\
 &= 1 + 12\cos^4 x - 12\cos^2 x + 3 \\
 &= 4 - 12\cos^2 x + 12\cos^4 x //
 \end{aligned}$$

$$\boxed{4(\cos^6 x + \sin^6 x) = 1 + 3\cos^2 2x}$$

OK!