

11.

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x^2) - f^2(x) \geq \frac{1}{4}$$

Seja  $x = 1$ .

Temos  $f(1^2) - f^2(1) \geq \frac{1}{4}$

$$f(1) - f^2(1) \geq \frac{1}{4}$$

$$\therefore 0 \geq \underbrace{f^2(1) - f(1) + \frac{1}{4}}_{\geq 0}$$

$$0 \geq \left(f(1) - \frac{1}{2}\right)^2$$

$$\therefore f(1) - \frac{1}{2} = 0$$

$$f(1) = \frac{1}{2} \quad (*)$$

Seja  $x = 0$ .

Temos  $f(0^2) - f^2(0) \geq \frac{1}{4}$

$$f(0) - f^2(0) \geq \frac{1}{4}$$

$$\therefore 0 \geq \underbrace{f^2(0) - f(0) + \frac{1}{4}}_{\geq 0}$$

$$0 \geq \left(f(0) - \frac{1}{2}\right)^2$$

