Cálculo B - Prova 1

Nome:

Resolva as integrais

1.0 **1.** $\int \frac{\sec^4(\ln x)}{x}\,dx$

1.5 **2.** $\int \frac{x^2-4x-4}{x^3-2x^2+4x-8}\,dx$

1.0 **3.** $\int \frac{1}{\sin x - \cos x + 2}\,dx$

1.0 **4.** $\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx$

1.0 **5.** Temos que $\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$. Explique se vale ou não a equação

$$\int_1^{\infty} \frac{1}{x(x+1)}\,dx = \int_1^{\infty} \frac{1}{x}\,dx - \int_1^{\infty} \frac{1}{x+1}\,dx$$

Tabela de Integrais

$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$

$\int \frac{1}{x}\,dx = \ln x$

$\int e^x\,dx = e^x$

$\int \sin x\,dx = -\cos x$

$\int \cos x\,dx = \sin x$

$\int \tan x\,dx = \ln|\sec x|$

$\int \cot x\,dx = -\ln|\csc x|$

$\int \sec x\,dx = \ln|\sec x + \tan x|$

$\int \csc x\,dx = \ln|\csc x - \cot x|$

$\int \sec^2 x\,dx = \tan x$

$\int \csc^2 x\,dx = -\cot x$

$\int \sec x\,\tan x\,dx = \sec x$

$\int \csc x\,\cot x\,dx = -\csc x$

Relações teis

$\sin mx \sin nx = \frac{1}{2}\cos(m-n)x - \frac{1}{2}\cos(m+n)x$

$\cos mx \cos nx = \frac{1}{2}\cos(m+n)x + \frac{1}{2}\cos(m-n)x$

$\sin mx \cos nx = \frac{1}{2}\sin(m-n)x + \frac{1}{2}\sin(m+n)x$

Substituição útil: $z = \tan\frac{x}{2}$, $\sin x = \frac{2z}{1+z^2}$, $\cos x = \frac{1-z^2}{1+z^2}$

Cálculo B - Prova 1

1. $\int \frac{\sec^4 \ln x}{x}\, dx = \int \sec^4 u \, du =$

$(u = \ln x \rightarrow du = \frac{1}{x}\, dx)$

$= \int \underbrace{\sec^2 u}\, \sec^2 u \, du$

$= \int (1 + tg^2 u) \sec^2 u \, du$

$= \int \sec^2 u \, du + \int tg^2 u \sec^2 u \, du$

$= tg\, u + \frac{tg^3 u}{3}$

$= tg(\ln x) + \frac{1}{3} tg^3(\ln x) + C$.

obs.

$\int \sec^4 u \, du = \frac{1}{3} \sec^2 u \; tg\, u + \frac{2}{3} tg\, u$

$= \frac{1}{3} (1 + tg^2 u)\, tg\, u + \frac{2}{3} tg\, u$

$= \frac{1}{3} tg\, u + \frac{1}{3} tg^3 u + \frac{2}{3} tg\, u$

$= tg\, u + \frac{1}{3} tg^3 u$

2. $\displaystyle\int \frac{x^2-4x-4}{x^3-2x^2+4x-8}\,dx$

$$\begin{array}{l|l} x^3-2x^2+4x-8 & x-2 \\ \cline{2-2} -x^3+2x^2 & x^2+4 \\ \hline \qquad 4x-8 & \\ \qquad -4x+8 & \\ \qquad\quad 0 & \end{array}$$

$$x^3-2x^2+4x-8 = (x-2)\underbrace{(x^2+4)}_{\text{irredutível}}$$

0.4

$$\frac{x^2-4x-4}{x^3-2x^2+4x-8} = \frac{x^2-4x-4}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4}$$

$$= \frac{A(x^2+4)+(Bx+C)(x-2)}{(x-2)(x^2+4)}$$

$\therefore$

$$x^2-4x-4 = A(x^2+4) + (Bx+C)(x-2)$$

$$= Ax^2+4A + Bx^2 - 2Bx + Cx - 2C$$

$$= (A+B)x^2 + (C-2B)x + 4A-2C$$

$\therefore$ $\begin{cases} A+B = 1 \\ C-2B = -4 \Rightarrow // C = -4+2B // \\ 4A-2C = -4 \Rightarrow 4A-2C = -4 \end{cases}$

$\therefore 4A+8-4B = -4$

$4A-4B = -12$

$/ A-B = -3 /$

$$\begin{cases} A+B=1 \\ A-B=-3 \end{cases} \quad \therefore \quad 2A=-2 \Rightarrow \boxed{A=-1}$$

$$\begin{cases} A+B=1 \\ \therefore \; -1+B=1 \end{cases} \quad \therefore \quad \boxed{B=2}$$

$$C=-4+2B=-4+2\cdot 2=0 \quad \therefore \quad \boxed{C=0}$$

0.4

$$\therefore \quad \frac{x^2-4x-4}{x^3-2x^2+4x-8} = \frac{-1}{x-2} + \frac{2x}{x^2+4}$$

$$\therefore \int \frac{x^2-4x-4}{x^3-2x^2+4x-8}\,dx = -\int \frac{dx}{x-2} + 2\int \frac{x}{x^2+4}\,dx$$

$$= -\ln|x-2| + \cancel{2}\,\frac{1}{\cancel{2}}\ln(x^2+4)$$

$$= \ln\left|\frac{x^2+4}{x-2}\right| + C$$

0.7

3. $\displaystyle\int \frac{1}{\sin x - \cos x + 2}\, dx =$

$$\begin{cases} z = \operatorname{tg}\frac{x}{2} \longrightarrow dz = \frac{1}{2}\sec^2\frac{x}{2}\, dx \\ \qquad\qquad\qquad\quad = \frac{1}{2}(1+z^2)\, dx \\ \qquad\qquad \therefore\ dx = \frac{2dz}{1+z^2} \end{cases}$$

$$= \int \frac{\frac{2dz}{1+z^2}}{\frac{2z}{1+z^2} - \frac{1-z^2}{1+z^2} + 2} = \int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{2z - 1 + z^2 + 2 + 2z^2}{\cancel{1+z^2}}}$$

$$= \int \frac{2dz}{3z^2 + 2z + 1}$$

$$= \frac{2}{3}\int \frac{dz}{z^2 + \frac{2}{3}z + \frac{1}{3}}$$

0.4

$$= \frac{2}{3}\int \frac{dz}{\left(z + \frac{1}{3}\right)^2 + \frac{2}{9}}$$

$$\left(z + \frac{1}{3} = \frac{\sqrt{2}}{3}\operatorname{tg}\theta \longrightarrow dz = \frac{\sqrt{2}}{3}\sec^2\theta\, d\theta \right)$$

$$= \frac{2}{3}\int \frac{\frac{\sqrt{2}}{3}\sec^2\theta\, d\theta}{\left(\frac{\sqrt{2}}{3}\right)^2 \operatorname{tg}^2\theta + \frac{2}{9}} = \frac{\cancel{2}}{\cancel{3}}\int \frac{\frac{\sqrt{2}}{\cancel{3}}\cancel{\sec^2\theta}\, d\theta}{\frac{\cancel{2}}{\cancel{9}}\cancel{\sec^2\theta}}$$

$$= \sqrt{2}\int d\theta = \sqrt{2}\,\theta = \sqrt{2}\,\operatorname{arctg}\frac{3}{\sqrt{2}}\left(z+\frac{1}{3}\right)$$

$$= \sqrt{2}\,\operatorname{arctg}\left[\frac{3}{\sqrt{2}}\left(\operatorname{tg}\frac{x}{2}+\frac{1}{3}\right)\right] + C.$$

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4. $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{x^2+1}\, dx =$

$$= \lim_{A \to -\infty} \int_A^C \frac{1}{x^2+1}\, dx + \lim_{B \to +\infty} \int_C^B \frac{1}{x^2+1}\, dx$$ 0.2

$$= \lim_{A \to -\infty} \operatorname{arctg} x \Big]_A^C + \lim_{B \to +\infty} \operatorname{arctg} x \Big]_C^B$$

$$= \lim_{A \to -\infty} \left(\operatorname{arctg} C - \operatorname{arctg} A\right) + \lim_{B \to +\infty} \left(\operatorname{arctg} B - \operatorname{arctg} C\right)$$

$$= \cancel{\operatorname{arctg} C} - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - \cancel{\operatorname{arctg} C}$$ 1.0

$$= \pi$$

5. $\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$

$$\int_1^{+\infty} \frac{1}{x(x+1)}\,dx = \lim_{t\to+\infty} \int_1^t \frac{1}{x(x+1)}\,dx$$

$$= \lim_{t\to+\infty} \int_1^t \left[\frac{1}{x} - \frac{1}{x+1}\right] dx$$

$$= \lim_{t\to+\infty} \left(\int_1^t \frac{1}{x}\,dx - \int_1^t \frac{1}{x+1}\,dx\right)$$

$$= \lim_{t\to+\infty} \left(\ln x\Big]_1^t - \ln(x+1)\Big]_1^t\right)$$

$$= \lim_{t\to+\infty} \left(\ln t - \cancel{\ln 1}^{\,0} - \ln(t+1) + \ln 2\right)$$

$$= \lim_{t\to+\infty} \left(\ln \frac{t}{t+1} + \ln 2\right)$$

Mas $\lim_{t\to+\infty} \ln \frac{t}{t+1} = \lim_{t\to\infty} \ln \frac{1}{1+\cancel{\frac{1}{t}}^{\,0}} = \ln 1 = 0$

$= \ln 2$

$\therefore \int_1^{+\infty} \frac{1}{x(x+1)}\,dx = \ln 2$ (*)

0.1

Mas

$$\int_1^{+\infty} \frac{1}{x}\,dx = \lim_{t\to+\infty} \int_1^{t} \frac{1}{x}\,dx$$

$$= \lim_{t\to+\infty} \ln x \Big]_1^t$$

$$= \lim_{t\to+\infty} \left(\ln t - \ln 1\right)$$

$$= +\infty$$

0.1

$$\int_1^{+\infty} \frac{1}{x+1}\,dx = \lim_{t\to+\infty} \int_1^{t} \frac{1}{x+1}\,dx$$

$$= \lim_{t\to+\infty} \ln(x+1) \Big|_1^t$$

$$= \lim_{t\to+\infty} \left(\ln(t+1) - \ln 2\right)$$

$$= +\infty$$

$$\therefore \int_1^{+\infty} \frac{1}{x}\,dx - \int_1^{+\infty} \frac{1}{x+1}\,dx = \infty - \infty$$

0.1

= diverge (**)

De (*) e (**) vemos que a igualdade proposta não se verifica.

0.7