

Cálculo B - Lista 13

Derivadas parciais

Encontre as derivadas parciais de primeira ordem das funções

1. $f(x, y) = \frac{2}{3}x^{\frac{3}{2}}$

2. $f(x, y) = 2x + 3x^2y^4$

3. $g(u, v) = \frac{u^3+v^3}{u^2+v^2}$

4. $f(x, y) = \sqrt{4 - x^2 - 9y^2}$

5. $z = \sqrt{(1 - x^{\frac{2}{3}})^3 - y^2}$

6. $z = (\sin(x^2y))^3$

7. $f(x, y, z) = x^2y^5 + xz^2$

8. $f(x, y, z) = \frac{x+y+z}{xy+yz+zx}$

9. $w = e^x(\cos y + \sin z)$

10. $w = \arcsin \frac{1}{1+xyz^2}$

Nos exercícios a seguir encontre as derivadas parciais f_{xx} , f_{xy} , f_{yx} da função $f(x, y)$.

11. $f(x, y) = 3x^2 - \sqrt{2}xy^2 + y^5 - 2$

12. $f(x, y) = x^2 - y^2$

13. $f(x, y) = \sqrt{x^2 + y^2}$

14. $f(x, y) = \frac{x}{x^2+y^2}$

Regra da cadeia

Nos exercícios a seguir use a regra da cadeia para calcular $\frac{dz}{dt}$, $\frac{dw}{dt}$

15. $z = 2x^2 - 3y^3$, $x = \sqrt{t}$, $y = e^{2t}$

16. $z = \sin x + \cos xy$, $x = t^2$, $y = t$

17. $z = \sqrt{2x - 4y}$, $x = \ln t$, $y = 1 - 3t^2$

18. $w = \frac{x}{y} - \frac{z}{x}$, $x = \sin t$, $y = \cos t$, $z = \tan t$

19. $w = \sqrt{x^2 + y^2 + z^2}$, $x = e^t$, $y = e^{-t}$, $z = 2t$

$$20. w = \sin xy^2z^3, \quad x = 3t, \quad y = \sqrt{t}, \quad z = \sqrt[3]{t}$$

$$21. w = e^{-x^2-y^2}, \quad x = t, \quad y = \sqrt{t}$$

$$22. w = \sin xyz, \quad x = t, \quad y = t^2, \quad z = t^3$$

Nos exercícios a seguir calcule $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$

$$23. z = \frac{4}{xy} - \frac{x}{y}, \quad x = u^2, \quad y = uv$$

$$24. z = \ln(x^2 - y^2), \quad x = u - v, \quad y = u^2 + v^2$$

$$25. z = \sin 2x \cos 3y, \quad x = (u + v)^2, \quad y = (u - v)^2$$

$$26. z = xe^y + ye^{-x}, \quad x = \ln u, \quad y = v \ln u$$

Nos exercícios a seguir dado $w = w(x, y, z)$ com $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$, calcule $\frac{\partial w}{\partial u}$ e $\frac{\partial w}{\partial v}$

$$27. w = \ln(x^2 + y^2 + z^2), \quad x = u - v, \quad y = u + v, \quad z = 2\sqrt{uv}$$

$$28. w = \sqrt{x^2 + y^2 + z^2}, \quad x = 3e^v \sin u, \quad y = 3e^v \cos u, \quad z = 4e^v$$

Derivada direcional

Para cada função $f(x, y)$ calcule a derivada direcional de f ao longo do vetor dado e avalie a derivada direcional no ponto P .

$$29. f(x, y) = x^2 + 2xy + 3y^2; \quad \vec{u} = (1, 1); \quad P = (2, 1)$$

$$30. f(x, y) = e^x \sin y; \quad \vec{u} = (1, -1); \quad P = (0, \pi/4)$$

$$31. f(x, y) = x^3 - x^2y + xy^2 + y^3; \quad \vec{u} = (2, 3); \quad P = (1, -1)$$

$$32. f(x, y) = \cot \frac{y}{x}; \quad \vec{u} = (3, 4); \quad P = (2, \pi)$$

$$33. f(x, y) = \sin x \cos y; \quad \vec{v} = (4, -3); \quad P = \left(\frac{\pi}{3}, -\frac{2\pi}{3}\right)$$

$$34. f(x, y) = 2x^2 - 3xy + y^2 + 15; \quad \vec{a} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right); \quad P = (1, 1)$$

$$35. f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}; \quad \vec{a} = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right); \quad P = (3, 4)$$

$$36. f(x, y) = e^{4y}; \quad \vec{a} = (4, 0); \quad P = \left(\frac{1}{2}, \frac{1}{4}\right)$$

Respostas

$$1. f_x = \sqrt{x}, \quad f_y = 0$$

$$2. f_x = 2 + 6xy^4, \quad f_y = 12x^2y^3$$

$$3. \quad g_u = \frac{u^4 + 3u^2v^2 - 2uv^3}{(u^2 + v^2)^2}, \quad g_v = \frac{v^4 + 3u^2v^2 - 2vu^3}{(u^2 + v^2)^2}$$

$$4. \quad f_x = \frac{-x}{\sqrt{4-x^2-9y^2}}, \quad f_y = \frac{-9y}{\sqrt{4-x^2-9y^2}}$$

5.

$$z_x = \frac{-(1-x^{\frac{2}{3}})^2}{x^{\frac{1}{3}}\sqrt{(1-x^{\frac{2}{3}})^3 - y^2}}, \quad z_y = \frac{-y}{\sqrt{(1-x^{\frac{2}{3}})^3 - y^2}}$$

$$6. \quad z_x = 6xy \sin^2(x^2y) \cos(x^2y), \quad z_y = 3x^2 \sin^2(x^2y) \cos(x^2y)$$

$$7. \quad f_x = 2xy^5 + z^2, \quad f_y = 5x^2y^4, \quad f_z = 2xz$$

8.

$$f_x = \frac{-y^2 - yz - z^2}{(xy + yz + xz)^2}, \quad f_y = \frac{-x^2 - xz - z^2}{(xy + yz + xz)^2}, \quad f_z = \frac{-x^2 - xy - y^2}{(xy + yz + xz)^2}$$

$$9. \quad w_x = e^x(\cos y + \sin z), \quad w_y = -e^x \sin y, \quad w_z = e^x \cos z$$

10.

$$w_x = \frac{yz^2}{\sqrt{1 - (xyz^2)^2}}, \quad w_y = \frac{xz^2}{\sqrt{1 - (xyz^2)^2}}, \quad w_z = \frac{2xyz}{\sqrt{1 - (xyz^2)^2}}$$

$$11. \quad f_x = 6x - \sqrt{2}y^2, \quad f_y = -2\sqrt{2}xy + 5y^4, \quad f_{xy} = f_{yx} = -2\sqrt{2}y$$

$$f_{xx} = 6, \quad f_{yy} = -2\sqrt{2}x + 20y^3$$

$$12. \quad f_x = 2x, \quad f_y = -2y, \quad f_{xy} = f_{yx} = 0, \quad f_{xx} = 2, \quad f_{yy} = -2$$

13.

$$f_x = \frac{x}{\sqrt{x^2 + y^2}}, \quad f_y = \frac{y}{\sqrt{x^2 + y^2}}, \quad f_{xy} = f_{yx} = \frac{-xy}{(x^2 + y^2)^{\frac{3}{2}}}$$

$$f_{xx} = \frac{y^2}{x^2 + y^2}, \quad f_{yy} = \frac{x^2}{x^2 + y^2}$$

14.

$$f_x = \frac{y^2 - x^2}{(x^2 + y^2)^2}, \quad f_y = \frac{-2xy}{(x^2 + y^2)^2}, \quad f_{xy} = f_{yx} = \frac{2y(3x^2 - y^2)}{(x^2 + y^2)^3}$$

$$f_{xx} = \frac{2x(x^2 - 3y^2)}{(x^2 + y^2)^3}, \quad f_{yy} = \frac{-2x(1 - 4y^2)}{(x^2 + y^2)^2}$$

$$15. \quad \frac{dz}{dt} = 2 - 18e^{6t}$$

$$16. \frac{dz}{dt} = 2t \cos t^2 - 3t^2 \sin t^3$$

17.

$$\frac{dz}{dt} = \frac{1 + 12t^2}{t\sqrt{2 \ln t - 4 - 12t^2}}$$

18.

$$\frac{dw}{dt} = \frac{\sin t \cos^2 t + \cos^2 t + \sin^3 t - 1}{\sin t \cos^2 t}$$

19.

$$\frac{dw}{dt} = \frac{e^{2t} - e^{-2t} + 4t}{\sqrt{e^{2t} + e^{-2t} + 4t^2}}$$

$$20. \frac{dw}{dt} = 9t^2 \cos(3t^3)$$

$$21. \frac{dw}{dt} = -(2t + 1)e^{-t^2-t}$$

$$22. \frac{dw}{dt} = 6t^5 \cos t^6$$

23.

$$\frac{\partial z}{\partial u} = \frac{-12}{u^4 v} - \frac{1}{v}, \quad \frac{\partial z}{\partial v} = \frac{-4}{u^3 v^2} + \frac{u}{v^2}$$

24.

$$\frac{\partial z}{\partial u} = \frac{2(u-v) - 4u(u^2 + v^2)}{(u-v)^2 - (u^2 + v^2)^2}$$

$$\frac{\partial z}{\partial v} = \frac{-2(u-v) - 4v(u^2 + v^2)}{(u-v)^2 - (u^2 + v^2)^2}$$

$$25. \frac{\partial z}{\partial u} = 4(u+v) \cos 2(u+v)^2 \cos 3(u-v)^2 - 6(u-v) \sin 2(u+v)^2 \sin 3(u-v)^2$$

$$\frac{\partial z}{\partial v} = 4(u+v) \cos 2(u+v)^2 \cos 3(u-v)^2 + 6(u-v) \sin 2(u+v)^2 \sin 3(u-v)^2$$

$$26. \frac{\partial z}{\partial u} = u^{v-1}(1 + v \ln u) + \frac{v}{u^2}(1 - \ln u)$$

$$\frac{\partial z}{\partial v} = (u^v \ln u + \frac{1}{u}) \ln u$$

$$27. \frac{\partial w}{\partial u} = \frac{2}{u+v}, \quad \frac{\partial w}{\partial v} = \frac{2}{u+v}$$

$$28. \frac{\partial w}{\partial u} = 0, \quad \frac{\partial w}{\partial v} = 5e^v$$

$$29. (D_{\hat{u}} f)(2, 1) = \frac{16}{\sqrt{2}}$$

$$30. (D_{\hat{u}} f)(0, \frac{\pi}{4}) = 0$$

$$31. (D_{\hat{u}}f)(1, -1) = \frac{12}{\sqrt{13}}$$

$$32. (D_{\hat{u}}f)(2, \pi) = \frac{3\pi-8}{20}$$

$$33. (D_{\hat{v}}f)\left(\frac{\pi}{3}, -\frac{2\pi}{3}\right) = -\frac{13}{20}$$

$$34. (D_{\hat{a}}f)(1, 1) = 0$$

$$35. (D_{\hat{a}}f)(3, 4) = \frac{96+72\sqrt{3}}{625}$$

$$36. (D_{\hat{a}}f)\left(\frac{1}{2}, \frac{1}{4}\right) = 0$$