

## Cálculo 2 - Lista 13

### Integração de funções trigonométricas envolvendo potências de seno e cosseno

Resolva cada uma das integrais a seguir

1.  $\int \sin^4 x \cos x \, dx$

2.  $\int \sin^5 x \cos x \, dx$

3.  $\int \cos^3 4x \sin 4x \, dx$

4.  $\int \cos^6 \frac{x}{2} \sin \frac{x}{2} \, dx$

5.  $\int \sin^3 x \, dx$

6.  $\int \sin^2 3x \, dx$

7.  $\int \sin^4 z \, dz$

8.  $\int \cos^5 x \, dx$

9.  $\int \cos^2 \frac{x}{2} \, dx$

10.  $\int \sin^3 x \cos^3 x \, dx$

11.  $\int \sin^2 x \cos^3 x \, dx$

12.  $\int \cos^6 x \, dx$

13.  $\int \sin^5 x \cos^2 x \, dx$

14.  $\int \sin^2 2t \cos^4 2t \, dt$

15.  $\int \sin^2 3t \cos^2 3t \, dt$

16.  $\int \sqrt{\cos z} \sin^3 z \, dz$

17.  $\int \frac{\cos^3 3x}{\sqrt[3]{\sin 3x}} \, dx$

18.  $\int \sin^3 \frac{y}{2} \cos^2 \frac{y}{2} \, dy$

19.  $\int \cos 4x \cos 3x \, dx$

20.  $\int \sin 2x \cos 4x \, dx$

21.  $\int \sin 3y \cos 5y \, dy$

22.  $\int \cos t \cos 3t \, dt$

23.  $\int (\sin 3t - \sin 2t)^2 \, dt$

24.  $\int \sin x \sin 3x \sin 5x \, dx$

25.  $\int 2 \sin x \cos x \, dx$

26. Mostre que  $\int_0^\pi \sin^2 nx \, dx = \frac{\pi}{2}$  para qualquer  $n$  inteiro positivo.

27. Mostre que  $\int_0^\pi \cos^n x \, dx = 0$  para qualquer  $n$  inteiro positivo ímpar.

28. Mostre que

$$\int_{-1}^1 \cos n\pi x \cos m\pi x \, dx = \begin{cases} 0, & m \neq n \\ 1, & m = n \end{cases}$$

29. Mostre que para  $n$  inteiro positivo ímpar tem-se a forma geral

$$\int \sin^n x \, dx = \sum_{r=0}^k \frac{k!}{(k-r)!r!} \frac{(-1)^{r+1}}{2r+1} \cos^{2r+1} x + C$$
$$\int \cos^n x \, dx = \sum_{r=0}^k \frac{k!}{(k-r)!r!} \frac{(-1)^r}{2r+1} \sin^{2r+1} x + C$$

### Resposta

1.  $\frac{1}{5} \sin^5 x + C$
2.  $\frac{1}{6} \sin^6 x + C$
3.  $-\frac{1}{16} \cos^4 4x + C$
4.  $-\frac{2}{7} \cos^7 \frac{x}{2} + C$
5.  $-\cos x + \frac{1}{3} \cos^3 x + C$
6.  $\frac{x}{2} - \frac{1}{12} \sin 6x + C$
7.  $\frac{3}{8}z - \frac{1}{4} \sin 2z + \frac{1}{32} \sin 4z + C$
8.  $\sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C$
9.  $\frac{1}{2}x + \frac{1}{2} \sin x + C$
10.  $-\frac{1}{16} \cos 2x + \frac{1}{48} \cos^3 2x + C$
11.  $\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$
12.  $\frac{5}{16}x + \frac{1}{4} \sin 2x + \frac{3}{64} \sin 4x - \frac{1}{48} \sin^3 2x + C$
13.  $-\frac{1}{3} \cos^3 x + \frac{2}{5} \cos^5 x - \frac{1}{7} \cos^7 x + C$
14.  $\frac{1}{16}t - \frac{1}{128} \sin 8t + \frac{1}{96} \sin^3 4t + C$
15.  $\frac{1}{8}t - \frac{1}{96} \sin 12t + C$
16.  $-\frac{2}{3} \cos^{3/2} z + \frac{2}{7} \cos^{7/2} z + C$
17.  $\frac{1}{2}(\sin 3x)^{2/3} - \frac{1}{8}(\sin 3x)^{8/3} + C$
18.  $-\frac{2}{3} \cos^3 \frac{y}{2} + \frac{2}{5} \cos^5 \frac{y}{2} + C$
19.  $\frac{1}{14} \sin 7x + \frac{1}{2} \sin x + C$
20.  $\frac{1}{4} \cos 2x - \frac{1}{12} \cos 6x + C$

21.  $\frac{1}{4} \cos 2y - \frac{1}{16} \cos 8y + C$

22.  $\frac{1}{8} \sin 4t + \frac{1}{4} \sin 2t + C$

23.  $t - \sin t - \frac{1}{8} \sin 4t + \frac{1}{5} \sin 5t - \frac{1}{12} \sin 6t + C$

24.  $-\frac{1}{12} \cos 3x - \frac{1}{28} \cos 7x + \frac{1}{4} \cos x + \frac{1}{36} \cos 9x + C$

25.  $\sin^2 x + C$