

Cálculo 2 - Lista 14

Integração de funções trigonométricas envolvendo potências de tangente, secante, cotangente e cosecante

Resolva cada uma das integrais a seguir

1. $\int \tan^2 5x \, dx$

2. $\int \cot^2 4t \, dt$

3. $\int x \cot^2 2x^2 \, dx$

4. $\int e^x \tan^2(e^x) \, dx$

5. $\int \cot^3 t \, dt$

6. $\int \tan^4 x \, dx$

7. $\int \tan^6 3x \, dx$

8. $\int \cot^5 2x \, dx$

9. $\int \sec^4 x \, dx$

10. $\int \csc^4 x \, dx$

11. $\int \csc^3 x \, dx$

12. $\int \sec^5 x \, dx$

13. $\int e^x \tan^4(e^x) \, dx$

14. $\int \frac{\sec^4(\ln x)}{x} \, dx$

15. $\int \tan^6 x \sec^4 x \, dx$

16. $\int \tan^5 x \sec^3 x \, dx$

17. $\int \cot^2 3x \csc^4 3x \, dx$

18. $\int (\sec 5x + \csc 5x)^2 \, dx$

19. $\int (\tan 2x + \cot 2x)^2 \, dx$

20. $\int \frac{1}{1+\cos x} \, dx$

21. $\int \frac{2\sin w - 1}{\cos^2 w} \, dw$

22. $\int \frac{\tan^3 \sqrt{x}}{\sqrt{x}} \, dx$

23. $\int \tan^5 3x \, dx$

24. $\int \frac{\tan^4 y}{\sec^5 y} dy$
25. $\int \frac{du}{1+\sec \frac{1}{2}u}$
26. $\int \frac{\csc^4 x}{\cot^2 x} dx$
27. $\int \frac{\sec^3 x}{\tan^4 x} dx$
28. $\int \frac{\sin^2 \pi x}{\cos^6 \pi x} dx$
29. $\int \frac{\tan^3(\ln x) \sec^6(\ln x)}{x} dx$
30. $\int \frac{\sec^4 w}{\sqrt{\tan w}} dw$
31. Mostre que $\int \cot x \csc^n x dx = -\frac{\csc^n x}{n} + C$ se $n \neq 0$.
32. Mostre que $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$ se $n \geq 1, n \in N$.
33. Deduza uma fórmula similar aquela do exercício (31) para $\int \tan x \sec^n x dx$, se $n \neq 0$.
34. Deduza uma fórmula similar aquela do exercício (32) para $\int \cot^n x dx$, se $n \geq 1, n \in N$.

Resposta

1. $\frac{\tan 5x}{5} - x + C$
2. $-\frac{1}{4} \cot 4t - t + C$
3. $-\frac{1}{4} \cot 2x^2 - \frac{1}{2}x^2 + C$
4. $\tan e^x - e^x + C$
5. $-\frac{1}{2} \cot^2 t + \ln |\csc t| + C$
6. $\frac{1}{3} \tan^3 x - \tan x + x + C$
7. $\frac{1}{15} \tan^5 3x - \frac{1}{9} \tan^3 3x + \frac{1}{3} \tan 3x - x + C$
8. $-\frac{1}{8} \cot^4 2x + \frac{1}{4} \cot^2 2x + \frac{1}{2} \ln |\sin 2x| + C$
9. $\tan x + \frac{1}{3} \tan^3 x + C$
10. $-\cot x - \frac{1}{3} \cot^3 x + C$
11. $-\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln |\csc x - \cot x| + C$
12. $\frac{1}{4} \sec^3 x \tan x + \frac{3}{8} \sec x \tan x + \frac{3}{8} \ln |\sec x + \tan x| + C$
13. $\frac{1}{3} \tan^3 e^x - \tan e^x + e^x + C$
14. $\tan \ln x + \frac{1}{3} \tan^3 \ln x + C$
15. $\frac{1}{7} \tan^7 x + \frac{1}{9} \tan^9 x + C$

16. $\frac{1}{7} \sec^7 x - \frac{2}{5} \sec^5 x + \frac{1}{3} \sec^3 x + C$
17. $-\frac{1}{15} \cot^5 3x - \frac{1}{9} \cot^3 3x + C$
18. $\frac{1}{5} \tan 5x - \frac{1}{5} \cot 5x + \frac{2}{5} \ln |\csc 10x - \cot 10x| + C$
19. $\frac{1}{2} \tan 2x - \frac{1}{2} \cot 2x + C$
20. $\tan \frac{x}{2} + C$
21. $2 \sec w - \tan w + C$
22. $\tan^2 \sqrt{x} - 2 \ln |\sec \sqrt{x}| + C$
23. $\frac{1}{12} \tan^4 3x - \frac{1}{6} \tan^2 3x + \frac{1}{3} \ln |\sec 3x| + C$
24. $\frac{1}{5} \sin^5 y + C$
25. $2(\cot \frac{u}{2} - \csc \frac{u}{2}) + u + C$
26. $-2 \cot 2x + C$
27. $-\frac{1}{3} \csc^3 x + C$
28. $\frac{1}{3\pi} \tan^3 \pi x + \frac{1}{5\pi} \tan^5 \pi x + C$
29. $\frac{1}{4} \tan^4 \ln x + \frac{1}{3} \tan^6 \ln x + \frac{1}{8} \tan^8 \ln x + C$
30. $2\sqrt{\tan w} + \frac{2}{5} \tan^{5/2} w + C$