Cálculo B - Lista 1

Integração de funções trigonométricas envolvendo potências de seno e cosseno

Resolva cada uma das integrais a seguir

1. $\int \sin^4 x \cos x \, dx$

2. $\int \sin^5 x \, \cos x \, dx$

3. $\int \cos^3 4x \, \sin 4x \, dx$

4. $\int \cos^6 \frac{x}{2} \, \sin \frac{x}{2} \, dx$

5. $\int \sin^3 x \, dx$

6. $\int \sin^2 3x \, dx$

7. $\int \sin^4 z \, dz$

8. $\int \cos^5 x \, dx$

9. $\int \cos^2 \frac{x}{2} \, dx$

10. $\int \sin^3 x \, \cos^3 x \, dx$

11. $\int \sin^2 x \, \cos^3 x \, dx$

12. $\int \cos^6 x \, dx$

13. $\int \sin^5 x \, \cos^2 x \, dx$

14. $\int \sin^2 2t \, \cos^4 2t \, dt$

15. $\int \sin^2 3t \, \cos^2 3t \, dt$

16. $\int \sqrt{\cos z} \, \sin^3 z \, dz$

17. $\int \frac{\cos^3 3x}{\sqrt[3]{\sin 3x}} \, dx$

18. $\int \sin^3 \frac{y}{2} \, \cos^2 \frac{y}{2} \, dy$

19. $\int \cos 4x \, \cos 3x \, dx$

20. $\int \sin 2x \, \cos 4x \, dx$

21. $\int \sin 3y \, \cos 5y \, dy$

22. $\int \cos t \, \cos 3t \, dt$

23. $\int (\sin 3t - \sin 2t)^2 \, dt$

24. $\int \sin x \, \sin 3x \, \sin 5x \, dx$

25. $\int 2 \sin x \, \cos x \, dx$

26. Mostre que $\int_0^\pi \sin^2 nx \, dx = \frac{\pi}{2}$ para qualquer n inteiro positivo.

27. Mostre que $\int_0^\pi \cos^n x \, dx = 0$ para qualquer n inteiro positivo ímpar.

28. Mostre que

$$\int_{-1}^1 \cos n\pi x \, \cos m\pi x \, dx = \begin{cases} 0 \, , & m \neq n \\ 1 \, , & m = n \end{cases}$$

29. Mostre que para n inteiro positivo ímpar tem-se a forma geral

$$\int \sin^n x \, dx \;=\; \sum_{r=0}^{k} \frac{k!}{(k-r)!r!} \frac{(-1)^{r+1}}{2r+1} \cos^{2r+1} x + C$$

$$\int \cos^n x \, dx \;=\; \sum_{r=0}^{k} \frac{k!}{(k-r)!r!} \frac{(-1)^r}{2r+1} \sin^{2r+1} x + C$$

Resposta

1. $\frac{1}{5} \sin^5 x + C$

2. $\frac{1}{6} \sin^6 x + C$

3. $-\frac{1}{16} \cos^4 4x + C$

4. $-\frac{2}{7} \cos^7 \frac{x}{2} + C$

5. $-\cos x + \frac{1}{3} \cos^3 x + C$

6. $\frac{x}{2} - \frac{1}{12} \sin 6x + C$

7. $\frac{3}{8} z - \frac{1}{4} \sin 2z + \frac{1}{32} \sin 4z + C$

8. $\sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C$

9. $\frac{1}{2} x + \frac{1}{2} \sin x + C$

10. $-\frac{1}{16} \cos 2x + \frac{1}{48} \cos^3 2x + C$

11. $\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$

12. $\frac{5}{16} x + \frac{1}{4} \sin 2x + \frac{3}{64} \sin 4x - \frac{1}{48} \sin^3 2x + C$

13. $-\frac{1}{3} \cos^3 x + \frac{2}{5} \cos^5 x - \frac{1}{7} \cos^7 x + C$

14. $\frac{1}{16} t - \frac{1}{128} \sin 8t + \frac{1}{96} \sin^3 4t + C$

15. $\frac{1}{8} t - \frac{1}{96} \sin 12t + C$

16. $-\frac{2}{3} \cos^{3/2} z + \frac{2}{7} \cos^{7/2} z + C$

17. $\frac{1}{2} (\sin 3x)^{2/3} - \frac{1}{8} (\sin 3x)^{8/3} + C$

18. $-\frac{2}{3} \cos^3 \frac{y}{2} + \frac{2}{5} \cos^5 \frac{y}{2} + C$

19. $\frac{1}{14} \sin 7x + \frac{1}{2} \sin x + C$

20. $\frac{1}{4} \cos 2x - \frac{1}{12} \cos 6x + C$

21. $\frac{1}{4}\cos 2y - \frac{1}{16}\cos 8y + C$

22. $\frac{1}{8}\sin 4t + \frac{1}{4}\sin 2t + C$

23. $t - \sin t - \frac{1}{8}\sin 4t + \frac{1}{5}\sin 5t - \frac{1}{12}\sin 6t + C$

24. $-\frac{1}{12}\cos 3x - \frac{1}{28}\cos 7x + \frac{1}{4}\cos x + \frac{1}{36}\cos 9x + C$

25. $\sin^2 x + C$