

Cálculo B - Lista 11

Derivadas parciais

Encontre as derivadas parciais de primeira ordem das funções

1. $f(x, y) = \frac{2}{3}x^{\frac{3}{2}}$

2. $f(x, y) = 2x + 3x^2y^4$

3. $g(u, v) = \frac{u^3+v^3}{u^2+v^2}$

4. $f(x, y) = \sqrt{4 - x^2 - 9y^2}$

5. $z = \sqrt{(1 - x^{\frac{2}{3}})^3 - y^2}$

6. $z = (\sin(x^2y))^3$

7. $f(x, y, z) = x^2y^5 + xz^2$

8. $f(x, y, z) = \frac{x+y+z}{xy+yz+zx}$

9. $w = e^x(\cos y + \sin z)$

10. $w = \arcsin \frac{1}{1+xyz^2}$

Nos exercícios a seguir encontre as derivadas parciais f_{xx} , f_{xy} , f_{zz} da função $f(x, y)$.

11. $f(x, y) = 3x^2 - \sqrt{2}xy^2 + y^5 - 2$

12. $f(x, y) = x^2 - y^2$

13. $f(x, y) = \sqrt{x^2 + y^2}$

14. $f(x, y) = \frac{x}{x^2+y^2}$

Regra da cadeia

Nos exercícios a seguir use a regra da cadeia para calcular $\frac{dz}{dt}$

15. $z = 2x^2 - 3y^3$, $x = \sqrt{t}$, $y = e^{2t}$

16. $z = \sin x + \cos xy$, $x = t^2$, $y = 1$

17. $z = \sqrt{2x - 4y}$, $x = \ln t$, $y = 1 - 3t^2$

18. $w = \frac{x}{y} - \frac{z}{x}$, $x = \sin t$, $y = \cos t$, $z = \tan t$

19. $w = \sqrt{x^2 + y^2 + z^2}$, $x = e^t$, $y = e^{-t}$, $z = 2t$

$$20. w = \sin xy^2 z^3, \quad x = 3t, \quad y = \sqrt{t}, \quad z = \sqrt[3]{t}$$

$$21. w = e^{-x^2 - y^2}, \quad x = t, \quad y = \sqrt{t}$$

$$22. w = \sin xyz, \quad x = t, \quad y = t^2, \quad z = t^3$$

Nos exercícios a seguir calcule $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$

$$23. z = \frac{4}{xy} - \frac{x}{y}, \quad x = u^2, \quad y = uv$$

$$24. z = \ln(x^2 - y^2), \quad x = u - v, \quad y = u^2 + v^2$$

$$25. z = \sin 2x \cos 3y, \quad x = (u + v)^2, \quad y = (u - v)^2$$

$$26. z = xe^y + ye^{-x}, \quad x = \ln u, \quad y = v \ln u$$

Nos exercícios a seguir dado $w = w(x, y, z)$ com $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$, calcule $\frac{\partial w}{\partial u}$ e $\frac{\partial w}{\partial v}$

$$27. w = \ln(x^2 + y^2 + z^2), \quad x = u - v, \quad y = u + v, \quad z = 2\sqrt{uv}$$

$$28. w = \sqrt{x^2 + y^2 + z^2}, \quad x = 3e^v \sin u, \quad y = 3e^v \cos u, \quad z = 4e^v$$

Derivada direcional

Para cada função $f(x, y)$ calcule a derivada direcional de f ao longo do vetor dado e avalie a derivada direcional no ponto P .

$$29. f(x, y) = x^2 + 2xy + 3y^2; \quad \vec{u} = (1, 1); \quad P = (2, 1)$$

$$30. f(x, y) = e^x \sin y; \quad \vec{u} = (1, -1); \quad P = (0, \pi/4)$$

$$31. f(x, y) = x^3 - x^2y + xy^2 + y^3; \quad \vec{u} = (2, 3); \quad P = (1, -1)$$

$$32. f(x, y) = \cot \frac{y}{x}; \quad \vec{u} = (3, 4); \quad P = (2, \pi)$$

$$33. f(x, y) = \sin x \cos y; \quad \vec{v} = (4, -3); \quad P = \left(\frac{\pi}{3}, -\frac{2\pi}{3}\right)$$

$$34. f(x, y) = 2x^2 - 3xy + y^2 + 15; \quad \vec{a} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right); \quad P = (1, 1)$$

$$35. f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}; \quad \vec{a} = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right); \quad P = (3, 4)$$

$$36. f(x, y) = e^{4y}; \quad \vec{a} = (4, 0); \quad P = \left(\frac{1}{2}, \frac{1}{4}\right)$$