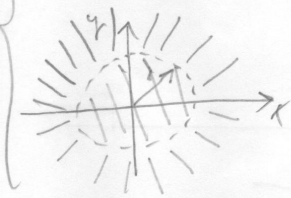
Cálculo B – Lista 10*

1. $f(x,y) = \dfrac{1}{x^2+y^2-1}$

$x^2+y^2-1 \neq 0$

$\begin{cases} x^2+y^2-1 = 0 \\ x^2+y^2 = 1 \end{cases}$ → Circunferência de raio 1

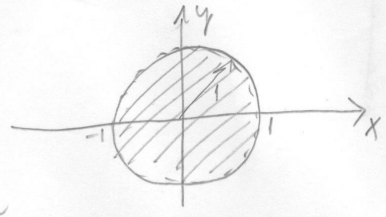
$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \neq 1\}$



2. $f(x,y) = \sqrt{1-x^2-y^2}$

$\begin{cases} 1-x^2-y^2 \geq 0 \\ 1 \geq x^2+y^2 \end{cases}$

$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$

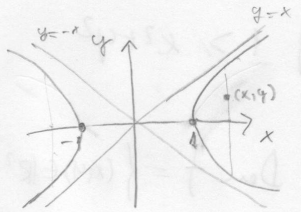


3. $f(x,y) = \sqrt{x^2-y^2-1}$

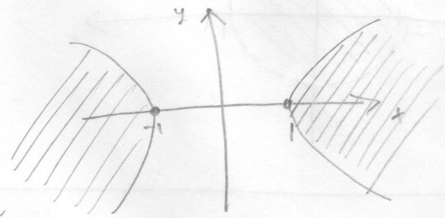
$\begin{cases} x^2-y^2-1 \geq 0 \\ x^2-y^2 \geq 1 \end{cases}$

$x^2-y^2 = 1$

$y = \pm\sqrt{x^2-1}$



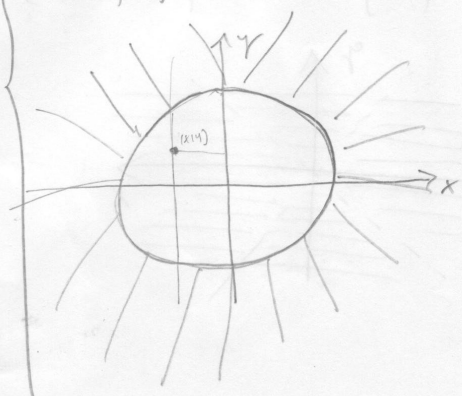
$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid x^2-y^2 \geq 1\}$



4. $f(x,y) = \sqrt{x^2+y^2-1}$

$\begin{cases} x^2+y^2-1 \geq 0 \\ x^2+y^2 \geq 1 \end{cases}$

$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \geq 1\}$

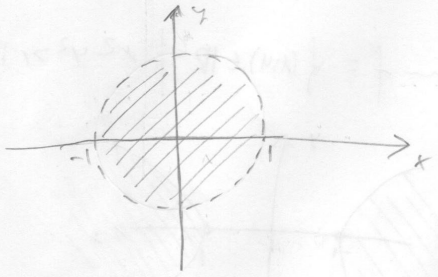


5. $f(x,y) = \dfrac{1}{\sqrt{1-x^2-y^2}}$

$1-x^2-y^2 > 0$

$1 > x^2+y^2$

$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 < 1\}$



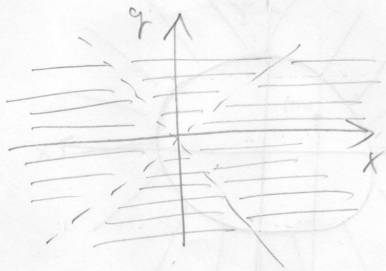
6. $f(x,y) = \dfrac{x^4-y^4}{x^2-y^2}$

$x^2-y^2 \neq 0$

$\therefore \; x^2-y^2 = 0$

$y \neq \pm x$

$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid y \neq \pm x\}$

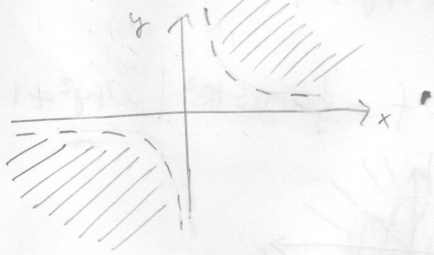


7. $f(x,y) = \ln(xy-1)$

$xy-1 > 0$

$xy > 1 \quad , x,y \neq 0$

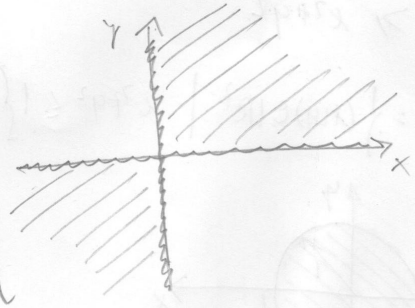
$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid xy > 1\}$



8. $f(x,y) = \sqrt{xy}$

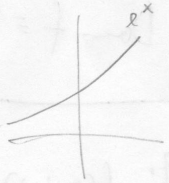
$xy \geq 0$

$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid xy \geq 0\}$



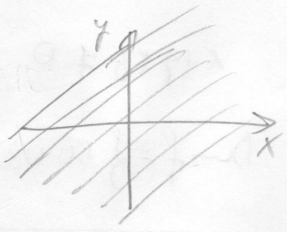
9. $f(x,y) = \dfrac{e^x - e^y}{e^x + e^y}$

$e^x > 0$, $e^y > 0$



$e^x + e^y \neq 0$, $\forall (x,y) \in \mathbb{R}^2$

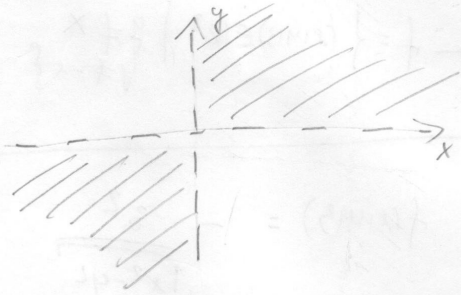
$\therefore \text{Dom } f = \mathbb{R}^2$



10. $f(x,y) = \ln(xy)$

$xy > 0$

$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid xy > 0\}$

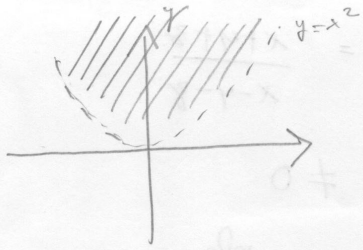


11. $f(x,y) = \dfrac{1}{\sqrt{y - x^2}}$

$y - x^2 > 0$

$y > x^2$

$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid y > x^2\}$



12. $f(x,y) = \sqrt{9 - x^2} - \sqrt{4 - y^2}$

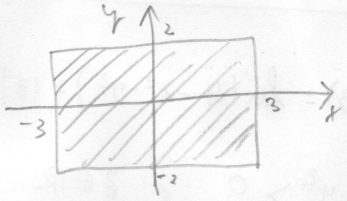
$9 - x^2 \geq 0$ e $4 - y^2 \geq 0$

$9 \geq x^2$ $\quad$ $4 \geq y^2$

$\therefore -3 \leq x \leq 3$ $\quad$ $-2 \leq y \leq 2$

$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid -3 \leq x \leq 3 \text{ e } -2 \leq y \leq 2\}$

$= [-3,3] \times [-2,2]$

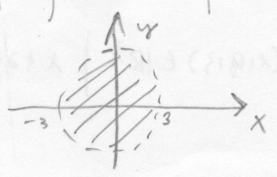


13. $f(x,y) = \dfrac{2}{\sqrt{9 - (x^2 + y^2)}}$

$9 - (x^2 + y^2) > 0$

$9 > x^2 + y^2$

$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 < 9\}$



14. $f(x,y,z) = \dfrac{x+y+z}{x-y-z}$

$\begin{cases} x-y-z \neq 0 \\ x-y-z = 0 \text{ plan} \end{cases}$

$\text{Dom } f = \{(x,y,z) \in \mathbb{R}^2 \mid x-y-z \neq 0\}$

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15. $f(x,y,z) = \sqrt{16-x^2-4y^2-z^2}$

$16-x^2-4y^2-z^2 \geq 0$

$x^2+4y^2+z^2 \leq 16$

$\text{Dom } f = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+4y^2+z^2 \leq 16\}$

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16. $f(x,y,z) = \ln(4-x^2-y^2) + |z|$

$\begin{cases} 4-x^2-y^2 > 0 \quad \& \quad z \in \mathbb{R} \\ 4 > x^2+y^2 \end{cases}$

$\text{Dom } f = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2 < 4\}$

---

17. $f(x,y,z) = \ln(x+2y+3z)$

$x+2y+3z > 0$

$\text{Dom } f = \{(x,y,z) \in \mathbb{R}^3 \mid x+2y+3z > 0\}$

---

18. $f(x,y,z) = \cos x + \cos y + \cos z$

$\text{Dom } f = \mathbb{R}^3$

---

19. $f(x,y,z) = \dfrac{x+y+z}{|x+y+z|}$

$x+y+z \neq 0$

$\text{Dom } f = \{(x,y,z) \in \mathbb{R}^3 \mid x+y+z \neq 0\}$

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20. $f(x,y,z) = \dfrac{z^2}{x^2-y^2}$

$x^2-y^2 \neq 0$

$x^2 \neq y^2$

$x \neq \pm y$

$\text{Dom } f = \{(x,y,z) \in \mathbb{R}^3 \mid \begin{matrix} y \neq x \\ y \neq -x \end{matrix}\}$

---

21. $f(x,y,z) = -\dfrac{z^2}{\sqrt{x^2-y^2}}$

$x^2-y^2 > 0$

$\text{Dom } f = \{(x,y,z) \in \mathbb{R}^3 \mid x^2-y^2 > 0\}$

22. $f(x,y,z) = \dfrac{\sqrt{1-x^2} + \sqrt{4-y^2}}{1+\sqrt{9-z^2}}$

$9 - z^2 \geq 0 \Rightarrow z^2 \geq 9$

$\therefore z \geq 3$ ou $z \leq -3$

$\text{Dom } f = \{(x,y,z) \in \mathbb{R}^3 \mid z \geq 3 \text{ ou } z \leq -3\}$

---

23. $f(x,y,z) = \ln(x+2y+3z)$

$x+2y+3z > 0$

$\text{Dom } f = \{(x,y,z) \in \mathbb{R}^3 \mid x+2y+3z > 0\}$

---

24. $f(x,y,z) = e^{\sqrt{4-x^2-y^2-z^2}}$

$4 - x^2 - y^2 - z^2 \geq 0$

$\text{Dom } f = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 \leq 4\}$

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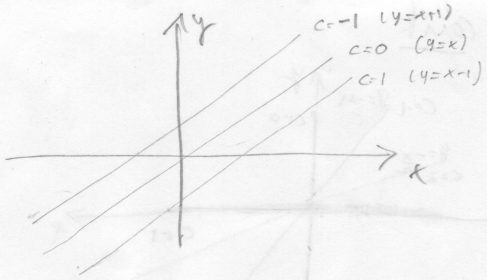
25. $f(x,y) = x - y$

$f(x,y) = c = x - y$

$\therefore y = x - c$: retas



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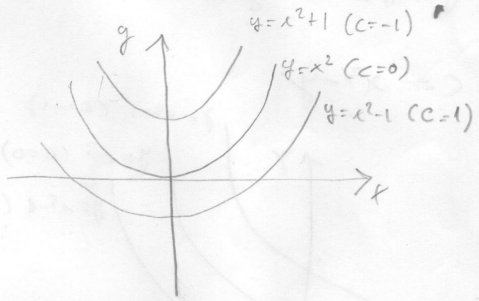
26. $f(x,y) = x^2 - y$

$f(x,y) = c = x^2 - y$



$1 = x^2 - y \rightarrow y = x^2 - 1$

$-1 = x^2 - y \rightarrow y = x^2 + 1$

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27. $f(x,y) = \dfrac{x}{x+y}$

$f(x,y) = c = \dfrac{x}{x+y}$

$c = 0 \Rightarrow x = 0$

$c = 1 \Rightarrow \dfrac{x}{x+y} = 1 \Rightarrow y = 0$

$c = -1 \Rightarrow \dfrac{x}{x+y} = -1 \Rightarrow y = -2x$

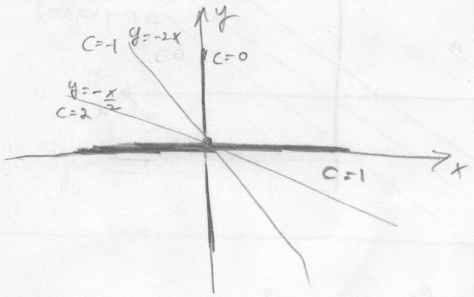
$c = 2 \Rightarrow \dfrac{x}{x+y} = 2 \Rightarrow x = 2x + 2y$

$\therefore y = -\dfrac{x}{2}$

$c = -2 \Rightarrow \dfrac{x}{x+y} = -2 \Rightarrow x = -2x - 2y$

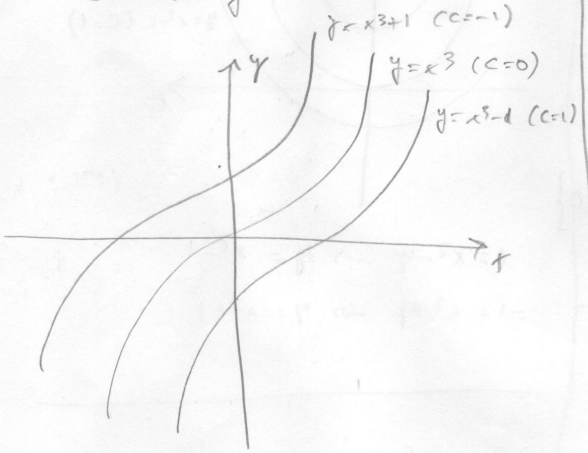
$y = -\dfrac{3x}{2}$

27. cont.



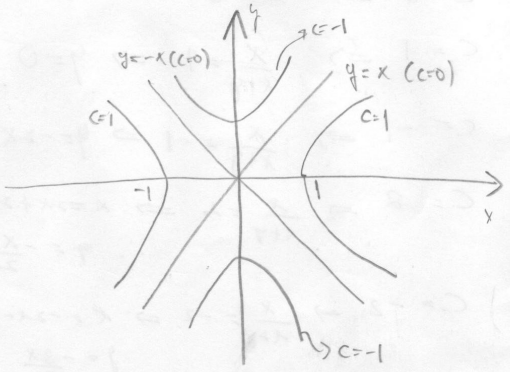
28. $f(x,y) = x^3 - y$

$c = x^3 - y$



29. $f(x,y) = x^2 - y^2$

$c = x^2 - y^2$



$c=1 : \quad 1 = x^2 - y^2$

$c=-1 : \quad -1 = x^2 - y^2$

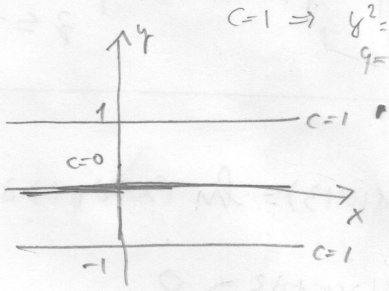
$1 = y^2 - x^2$

30. $f(x,y) = y^2$

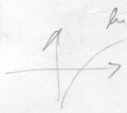
$c = y^2$ $\quad c=0 \Rightarrow y=0, \ x \in \mathbb{R}$

$c=1 \Rightarrow y^2 = 1, \ x \in \mathbb{R}$

$y = \pm 1$



31. $f(x,y) = \ln(x^2+y^2)$



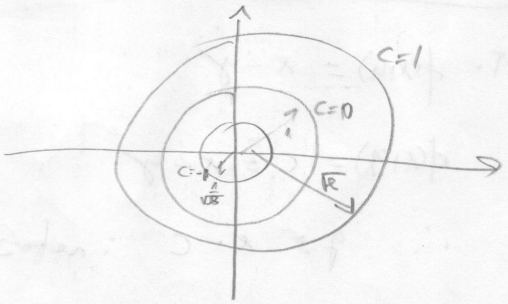
$c = \ln(x^2+y^2)$

$\therefore \quad x^2+y^2 = e^c$

$c=0 \Rightarrow x^2+y^2 = 1$

$c=1 \Rightarrow x^2+y^2 = e$

$c=-1 \Rightarrow x^2+y^2 = \frac{1}{e}$



32. $f(x,y) = \dfrac{\ln y}{x^2}$

$c = \dfrac{\ln y}{x^2}$

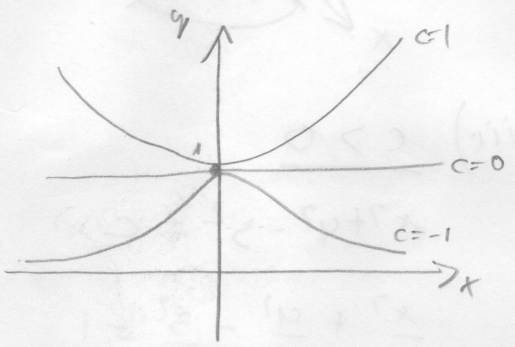
$\ln y = cx^2$

$y = e^{cx^2}$

$c=0 : y = e^0 = 1, \quad x \in \mathbb{R}$

$c=1 : y = e^{x^2}$

$c=-1 : y = e^{-x^2}$



$y = e^{x^2}, \quad y' = 2x e^{x^2}, \quad y'' = 2e^{x^2} + 4x^2 e^{x^2}$

33. $f(x,y) = \dfrac{x^2}{x^2+y^2} \quad ; (x,y) \neq (0,0)$

$c = \dfrac{x^2}{x^2+y^2} \qquad 0 \leq c \leq 1$

$cx^2 + cy^2 = x^2$

$cy^2 = x^2(1-c)$

$y^2 = \dfrac{1-c}{c} x^2$

$y = \pm \sqrt{\dfrac{1-c}{c}}\, x \qquad c > 0$

Se $c=0$:

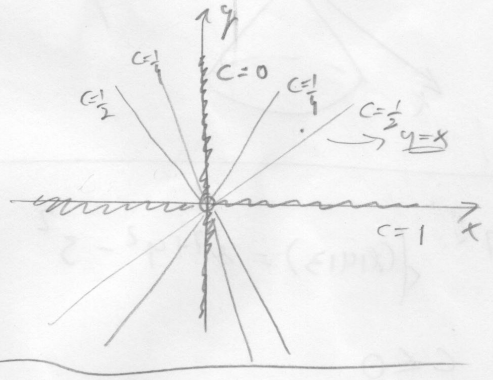
$0 = \dfrac{x^2}{x^2+y^2}$

$\Rightarrow x^2 = 0 \Rightarrow x = 0, \; y \in \mathbb{R}, \; y \neq 0$

Se $c=1$

$1 = \dfrac{x^2}{x^2+y^2}$

$x^2 + y^2 = x^2 \Rightarrow y = 0, \; x \in \mathbb{R}, \; x \neq 0$



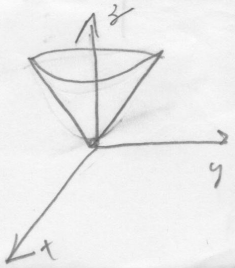
34. omitido

35. $\begin{cases} f(x,y,z) = 3(x^2+y^2)^{-1/2} \\ c = 1 \end{cases}$

$3(x^2+y^2)^{-1/2} = 1 \quad ; (x,y) \neq (0,0)$

$z = \sqrt{x^2+y^2}$

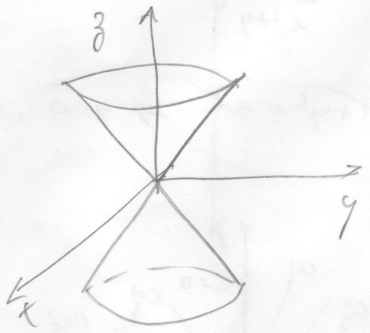
$\begin{cases} z^2 = x^2 + y^2 \\ z \geq 0 \end{cases}$



36. $\begin{cases} f(x,y,z) = 4x^2+9y^2-72z \\ c=0 \end{cases}$

$4x^2+9y^2-72z=0$

$z = \frac{x^2}{18} + \frac{y^2}{8}$



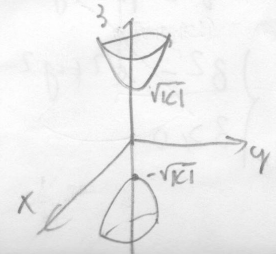
37. $f(x,y,z) = x^2+y^2-z^2$

i) $c<0$

$f(x,y,z) = x^2+y^2-z^2 = -|c|$

$\therefore \frac{x^2}{|c|} + \frac{y^2}{|c|} - \frac{z^2}{|c|} = -1$

{ hiperboloide de 2 folhas com vértices em $(0,0,\pm\sqrt{|c|})$

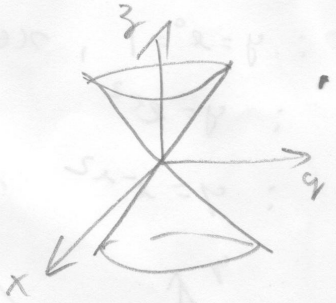


ii) $c=0$

$x^2+y^2-z^2=0$

$\therefore z^2 = x^2+y^2$

{ Cone com vértice na origem

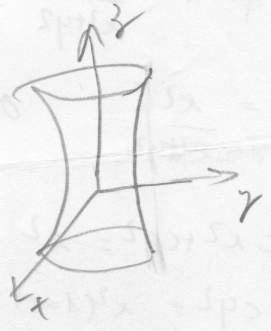


iii) $c>0$

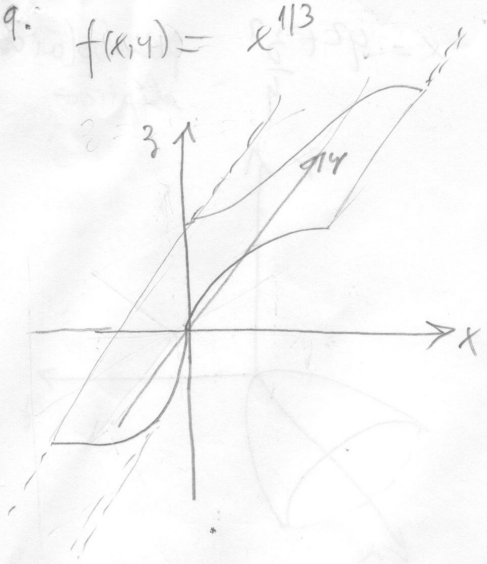
$x^2+y^2-z^2 = c$

$\frac{x^2}{c} + \frac{y^2}{c} - \frac{z^2}{c} = 1$

{ hiperboloide de 1 folha



39. $f(x,y) = x^{1/3}$

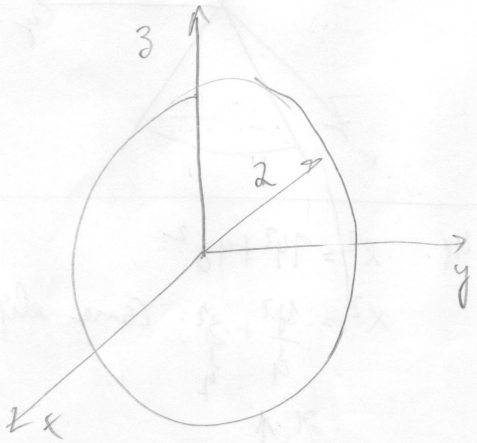


40. $f(x,y) = \sqrt{4-x^2-y^2}$

$z = \sqrt{4-x^2-y^2}$

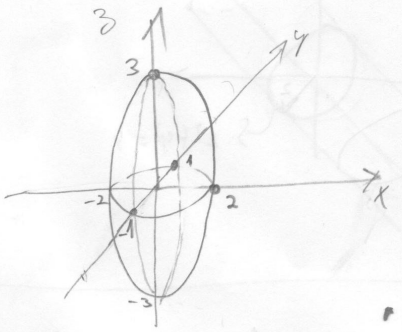
$z^2 = 4-x^2-y^2$

$x^2+y^2+z^2 = 4$



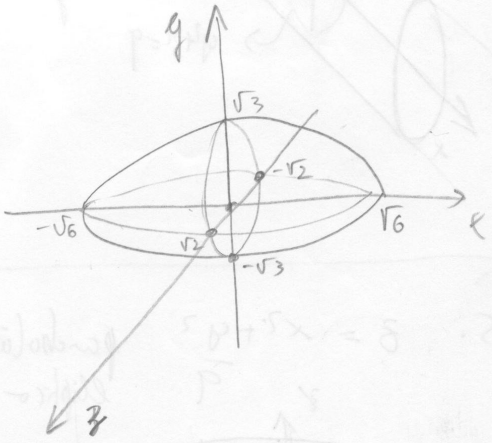
41. $\frac{x^2}{4} + \frac{y^2}{1} + \frac{z^2}{9} = 1$

Elipsóide

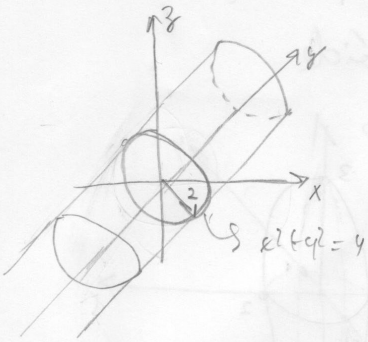


42. $x^2 + 2y^2 + 3z^2 = 6$

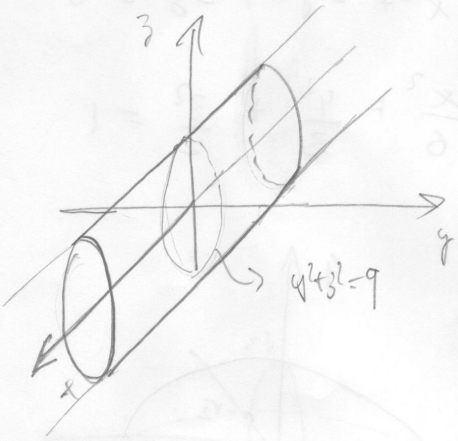
$\frac{x^2}{6} + \frac{y^2}{3} + \frac{z^2}{2} = 1$



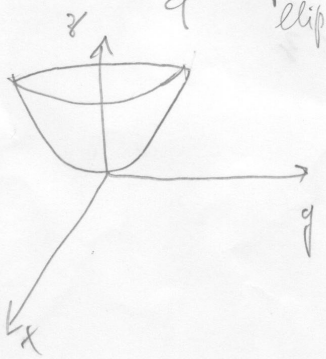
43. $x^2 + z^2 = 4$ cilindro ao longo do eixo y



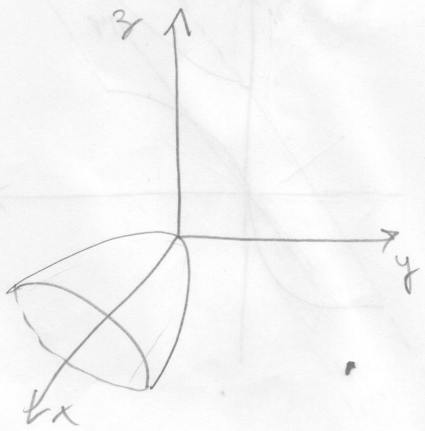
44. $y^2 + z^2 = 9$ cilindro ao longo do eixo x



45. $z = \frac{x^2}{9} + \frac{y^2}{9}$ parabolóide elíptico

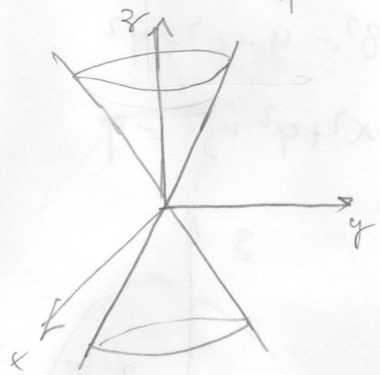


46. $x = \frac{y^2}{4} + \frac{z^2}{4}$ parabolóide elíptico



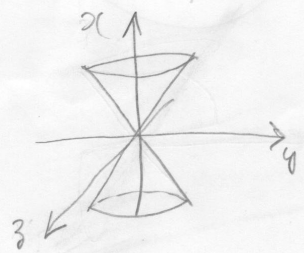
47. $z^2 = x^2 + 4y^2$

$z^2 = x^2 + \frac{y^2}{\frac{1}{4}}$ cone elíptico

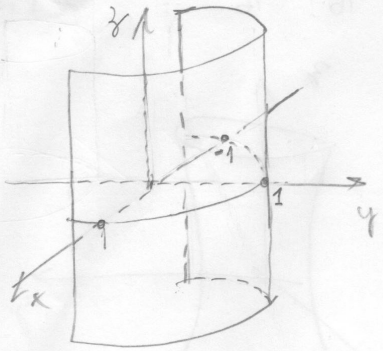


48. $x^2 = 9y^2 + 9z^2$

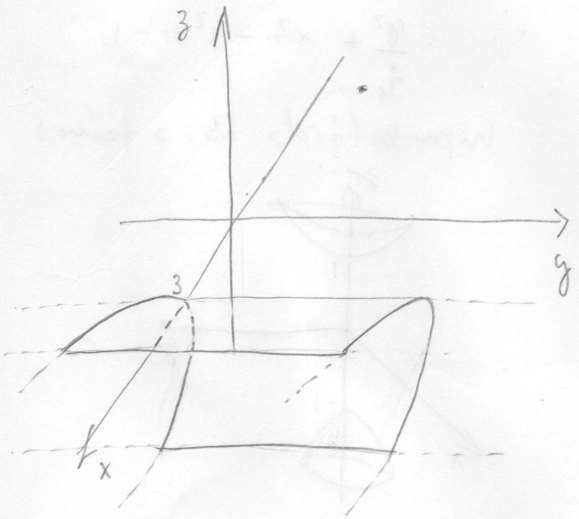
$x^2 = \frac{y^2}{\frac{1}{9}} + \frac{z^2}{\frac{1}{9}}$ cone elíptico



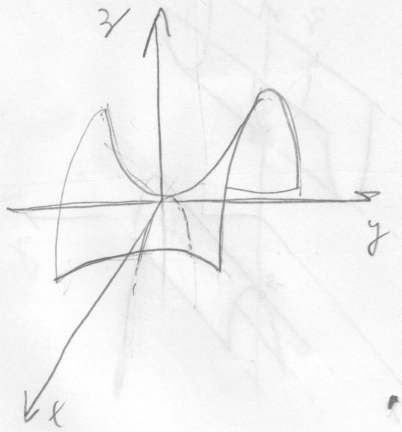
49. $y = 1 - x^2$ Cilindro parabólico



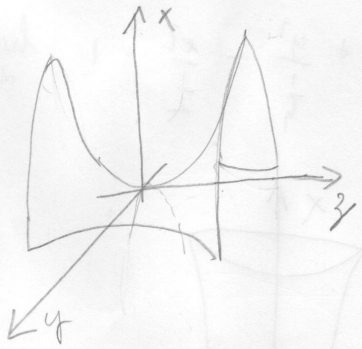
50. $x = z^2 + 3$ cilíndrico parabólico



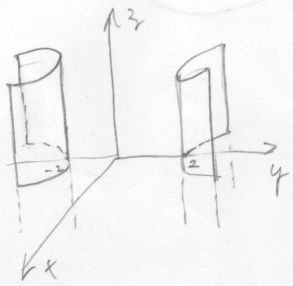
51. $z = y^2 - 4x^2$ Paraboloide hiperbólico



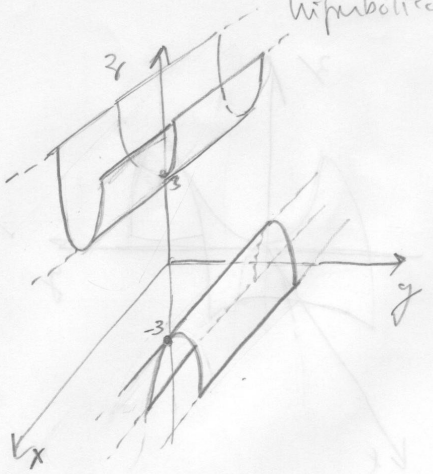
52. $x = 4z^2 - y^2$



53. $y^2 - x^2 = 4$ Cilindro hiperbólico

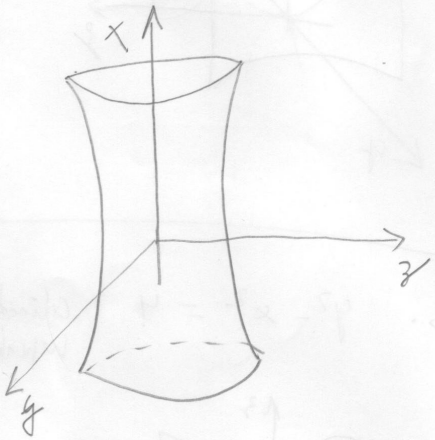


54. $z^2 - y^2 = 4$ Cilindro hiperbólico



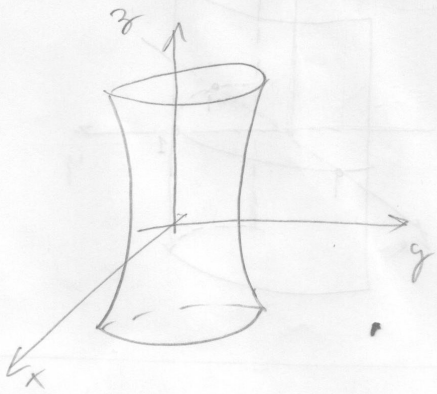
55. $z^2 + 4y^2 - 2x^2 = 1$

$\therefore \quad z^2 + \frac{y^2}{\frac{1}{4}} - \frac{x^2}{\frac{1}{2}} = 1$ hiperbolóide de 1 folha



56. $4x^2 + y^2 - z^2 = 16$

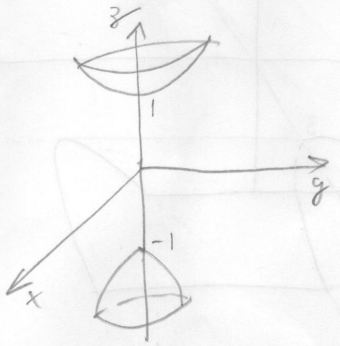
$\frac{x^2}{4} + \frac{y^2}{16} - \frac{z^2}{16} = 1$ hiperbolóide de 1 folha



57. $z^2 - 4y^2 - x^2 = 1$

$\therefore \quad \frac{z^2}{1} + x^2 - \frac{y^2}{\frac{1}{4}} = -1$

hiperbolóide de 2 folhas



58. $x^2 - 9y^2 - 4z^2 = 36$

$\therefore \quad \frac{y^2}{4} + \frac{z^2}{9} - \frac{x^2}{36} = -1$

hiperbolóide de 2 folhas