

## Cálculo C - Lista 15

### Transformada de Laplace (I)

#### Propriedades:

$$\mathcal{L}(f') = s\mathcal{L}(f) - f(0) \quad (1)$$

$$\mathcal{L}(f^{(n)}) = s^n \mathcal{L}(f) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0) \quad (2)$$

$$\mathcal{L}\left(\int_0^t f(u) du\right) = \frac{1}{s} \mathcal{L}(f) \quad (3)$$

Encontre a transformada de Laplace das seguintes funções

1.  $t + 4$

2.  $a + bt + ct^2$

3.

4.

5.

$$f(t) = \begin{cases} 0, & 0 \leq t < 1 \\ t, & 1 \leq t < 2 \\ 1, & 2 \leq t \end{cases}$$

6.

$$f(t) = \begin{cases} 0, & 0 \leq t \leq 5 \\ e^{-t}, & 5 < t \end{cases}$$

7.  $(t^2 + \frac{1}{2})^2$

8.  $\sin \pi t$

9.  $\cos(\omega t + \theta)$

10.  $\cos^2 t$

11.  $\cos^2 \omega t$

12.  $\sinh^2 2t$

Use os teoremas da derivada (1) e (2) e obtenha as transformadas a seguir

13.  $\mathcal{L}(t \cos \omega t) = \frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$

14.  $\mathcal{L}(t \sin \omega t) = \frac{2\omega s}{(s^2 + \omega^2)^2}$

15.  $\mathcal{L}(t \cosh at) = \frac{s^2 + a^2}{(s^2 - a^2)^2}$

16.  $\mathcal{L}(t \sinh at) = \frac{2as}{(s^2 - a^2)^2}$

Use o teorema da integral (3) para calcular  $f(t)$  sendo dado  $\mathcal{L}(f)$

17.  $\frac{1}{s^2 + s}$

18.  $\frac{10}{s(s^2 + 9)}$

19.  $\frac{1}{s^2(s+1)}$

20.  $\frac{1}{s^2} \left( \frac{s-1}{s+1} \right)$

21.  $\frac{54}{s^3(s-3)}$

22.  $\frac{1}{s^2} \left( \frac{s+1}{s^2+1} \right)$