Geometria Analítica - Prova 1

Nome:

Matrícula:

1. O determinante da matriz A

$$A = \begin{pmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+z & 1 \\ 1 & 1 & 1 & 1-z \end{pmatrix} \quad \text{é}$$

(A) 0 (B) $x+z$ (C) $(1-x^2)(1-z^2)$ (D) $4-xz$ (E) x^2z^2 (F) x^2+z^2

2. Se a, b, c, d não são nulos então

$$\det \begin{pmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+acd \\ 1 & c & c^2 & c^3+abd \\ 1 & d & d^2 & d^3+abc \end{pmatrix} \quad \text{é}$$

(A) 0 (B) $(abcd)^3$ (C) $2ab+2ac+2ad+2bc+2bd+2cd$
(D) $(b-a)(c-a)(d-a)(c-b)(d-b)(d-c)$ (E) $abcd$ (F) $a^2+b^2+c^2+d^2$

3. Sejam $A = (a_{ij})_{4\times 7}$ e $B = (b_{ij})_{7\times 9}$ matrizes dadas por $a_{ij} = i - j$, $b_{ij} = i$. Seja $C = (c_{ij}) = AB$. O elemento c_{33} é

(A) 2 (B) -1 (C) -56 (D) 0 (E) -34 (F) 18

4. Para que valores reais de m e n o sistema é indeterminado?

$$\begin{cases} x + my - z = 1 \\ 2x - y + z = n \\ 3x + y - 2z = 2n \end{cases}$$

(A) $m \neq 4/7,\ n \neq 7/5$
(B) $m \neq 4/7,\ n = 7/5$
(C) $m = 4/7,\ n \neq 7/5$
(D) $m = 4/7,\ n = 7/5$
(E) $m = 7/4,\ n = 5/7$
(F) $m \neq 7/4,\ n \neq 5/7$

5. Marque as sentenças em verdadeiro (V) ou falso (F)

(F) $\det(A+B) = \det A + \det B$ quaisquer que sejam as matrizes A e B.
(F) Se $AB = AC$ então $B = C$.
(F) Se $AB = 0$ então $A = 0$ ou $B = 0$, onde 0 denota a matriz nula.
(V) $(AB)C = A(BC)$

1.

$$\det\begin{bmatrix}1+x & 1 & 1 & 1\\ 1 & 1-x & 1 & 1\\ 1 & 1 & 1+z & 1\\ 1 & 1 & 1 & 1-z\end{bmatrix} = \det\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 1-x & 1 & 1\\ 1 & 1 & 1+z & 1\\ 1 & 1 & 1 & 1-z\end{bmatrix} + \det\begin{bmatrix}x & 1 & 1 & 1\\ 0 & 1-x & 1 & 1\\ 0 & 1 & 1+z & 1\\ 0 & 1 & 1 & 1-z\end{bmatrix} =$$

$$= \det\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\\ 1 & 1 & 1+z & 1\\ 1 & 1 & 1 & 1-z\end{bmatrix}^{\nearrow 0} + \det\begin{bmatrix}1 & 0 & 1 & 1\\ 1 & -x & 1 & 1\\ 1 & 0 & 1+z & 1\\ 1 & 0 & 1 & 1-z\end{bmatrix} + \det\begin{bmatrix}x & 1 & 1 & 1\\ 0 & 1 & 1 & 1\\ 0 & 1 & 1+z & 1\\ 0 & 1 & 1 & 1-z\end{bmatrix} + \det\begin{bmatrix}x & 0 & 1 & 1\\ 0 & -x & 1 & 1\\ 0 & 0 & 1+z & 1\\ 0 & 0 & 1 & 1-z\end{bmatrix}$$

$$= \det\begin{bmatrix}1 & 0 & 1 & 1\\ 1 & -x & 1 & 1\\ 1 & 0 & 1 & 1\\ 1 & 0 & 1 & 1-z\end{bmatrix}^{\nearrow 0} + \det\begin{bmatrix}1 & 0 & 0 & 1\\ 1 & -x & 0 & 1\\ 1 & 0 & z & 1\\ 1 & 0 & 0 & 1-z\end{bmatrix} + \det\begin{bmatrix}x & 1 & 1 & 1\\ 0 & 1 & 1 & 1\\ 0 & 1 & 1 & 1\\ 0 & 1 & 1 & 1-z\end{bmatrix}^{\nearrow 0} + \det\begin{bmatrix}x & 1 & 0 & 1\\ 0 & 1 & 0 & 1\\ 0 & 1 & z & 1\\ 0 & 1 & 0 & 1-z\end{bmatrix} +$$

$$+ \det\begin{bmatrix}x & 0 & 1 & 1\\ 0 & -x & 1 & 1\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & 1-z\end{bmatrix} + \det\begin{bmatrix}x & 0 & 0 & 1\\ 0 & -x & 0 & 1\\ 0 & 0 & z & 1\\ 0 & 0 & 0 & 1-z\end{bmatrix}$$

$$= \det\begin{bmatrix}1 & 0 & 0 & 1\\ 1 & -x & 0 & 1\\ 1 & 0 & z & 1\\ 1 & 0 & 0 & 1\end{bmatrix}^{\nearrow 0} + \det\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & -x & 0 & 0\\ 1 & 0 & z & 0\\ 1 & 0 & 0 & -z\end{bmatrix} + \det\begin{bmatrix}x & 1 & 0 & 1\\ 0 & 1 & 0 & 1\\ 0 & 1 & z & 1\\ 0 & 1 & 0 & 1\end{bmatrix}^{\nearrow 0} + \det\begin{bmatrix}x & 1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 1 & z & 0\\ 0 & 1 & 0 & -z\end{bmatrix}$$

$$+ \det\begin{bmatrix}x & 0 & 1 & 1\\ 0 & -x & 1 & 1\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & 1\end{bmatrix}^{\nearrow 0} + \det\begin{bmatrix}x & 0 & 1 & 0\\ 0 & -x & 1 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 1 & -z\end{bmatrix} + \det\begin{bmatrix}x & 0 & 0 & 1\\ 0 & -x & 0 & 1\\ 0 & 0 & z & 1\\ 0 & 0 & 0 & 1-z\end{bmatrix}$$

$$\det A = \det\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -x & 0 & 0 \\ 1 & 0 & z & 0 \\ 1 & 0 & 0 & -z \end{bmatrix} + \det\begin{bmatrix} x & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & z & 0 \\ 0 & 1 & 0 & -z \end{bmatrix} + \det\begin{bmatrix} x & 0 & 1 & 0 \\ 0 & -x & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -z \end{bmatrix} + \det\begin{bmatrix} x & 0 & 0 & 1 \\ 0 & -x & 0 & 1 \\ 0 & 0 & z & 1 \\ 0 & 0 & 0 & 1-z \end{bmatrix}$$

$\downarrow L_3 \to L_3 - L_4$ $\qquad\qquad$ $\downarrow L_4 \to L_4 - L_3$

$$= xz^2 + \det\begin{bmatrix} x & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & z & z \\ 0 & 1 & 0 & -z \end{bmatrix} + \det\begin{bmatrix} x & 0 & 1 & 0 \\ 0 & -x & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -z \end{bmatrix} - x^2 z(1-z)$$

$\downarrow L_4 \to L_4 - L_2$

$$= xz^2 + \det\begin{bmatrix} x & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & z & z \\ 0 & 0 & 0 & -z \end{bmatrix} + x^2 z - x^2 z + x^2 z^2$$

$$= \cancel{xz^2} - \cancel{xz^2} + \cancel{x^2 z} - \cancel{x^2 z} + x^2 z^2$$

$$= x^2 z^2$$

2.

$$\det\begin{bmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+acd \\ 1 & c & c^2 & c^3+abd \\ 1 & d & d^2 & d^3+abc \end{bmatrix} \xrightarrow[\substack{L_3 \to L_3 - L_1 \\ L_4 \to L_4 - L_1}]{L_2 \to L_2 - L_1} \det\begin{bmatrix} 1 & a & a^2 & a^3+bcd \\ 0 & b-a & b^2-a^2 & b^3-a^3-cd(b-a) \\ 0 & c-a & c^2-a^2 & c^3-a^3-bd(c-a) \\ 0 & d-a & d^2-a^2 & d^3-a^3-bc(d-a) \end{bmatrix}$$

$$= \det\begin{bmatrix} b-a & b^2-a^2 & b^3-a^3-cd(b-a) \\ c-a & c^2-a^2 & c^3-a^3-bd(c-a) \\ d-a & d^2-a^2 & d^3-a^3-bc(d-a) \end{bmatrix} =$$

$$= \det\begin{bmatrix} b-a & (b-a)(b+a) & (b-a)(b^2+ab+a^2-cd) \\ c-a & (c-a)(c+a) & (c-a)(c^2+ac+a^2-bd) \\ d-a & (d-a)(d+a) & (d-a)(d^2+ad+a^2-bc) \end{bmatrix}$$

$$= (b-a)(c-a)(d-a)\det\begin{bmatrix} 1 & b+a & b^2+ab+a^2-cd \\ 1 & c+a & c^2+ac+a^2-bd \\ 1 & d+a & d^2+ad+a^2-bc \end{bmatrix}$$

$$\Big\downarrow \begin{array}{l} L_2 \to L_2 - L_1 \\ L_3 \to L_3 - L_1 \end{array}$$

$$= (b-a)(c-a)(d-a)\det\begin{bmatrix} 1 & b+a & b^2+ab+a^2-cd \\ 0 & c-b & c^2+ac-bd-b^2-ab+cd \\ 0 & d-b & d^2+ad-bc-b^2-ab+cd \end{bmatrix}$$

$$= (b-a)(c-a)(d-a) \det \begin{bmatrix} 1 & b+a & b^2+ab+a^2-cd \\ 0 & (c-b) & (c-b)(c+b)+a(c-b)+d(c-b) \\ 0 & d-b & (d-b)(d+b)+a(d-b)+c(d-b) \end{bmatrix}$$

$$= (b-a)(c-a)(d-a) \det \begin{bmatrix} 1 & b+a & b^2+ab+a^2-cd \\ 0 & (c-b) & (c-b)[c+b+a+d] \\ 0 & (d-b) & (d-b)[d+b+a+c] \end{bmatrix}$$

$$= (b-a)(c-a)(d-a)(c-b)(d-b) \underbrace{\det \begin{bmatrix} 1 & b+a & b^2+ab+a^2-cd \\ 0 & 1 & a+b+c+d \\ 0 & 1 & a+b+c+d \end{bmatrix}}_{0}$$

$$= 0$$

3. $\begin{cases} A=(a_{ij})_{4\times 7} \\ a_{ij}=i-j \end{cases}$, $\begin{cases} B=(b_{ij})_{7\times 9} \\ b_{ij}=i \end{cases}$, $\begin{cases} C=(AB)_{4\times 9} \\ C=(c_{il}) \end{cases}$

$$c_{il}=\sum_{j=1}^{7} a_{ij}\, b_{jl}$$

$$c_{33}=\sum_{j=1}^{7} a_{3j}\, b_{j3} = a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33}+a_{34}b_{43}+a_{35}b_{53}+a_{36}b_{63}+a_{37}b_{73}$$

$$= 2\cdot 1 + 1\cdot 2 + 0\cdot 3 + (-1)\cdot 4 + (-2)\cdot 5 + (-3)\cdot 6 + (-4)\cdot 7$$

$$= \cancel{2}+\cancel{2}-\cancel{4}-10-18-28$$

$$= -56$$

4. $\begin{cases} x + my - z = 1 \\ 2x - y + z = n \\ 3x + y - 2z = 2n \end{cases}$

$$\tilde{A} = \begin{bmatrix} 1 & m & -1 & 1 \\ 2 & -1 & 1 & n \\ 3 & 1 & -2 & 2n \end{bmatrix} \xrightarrow{L_2 \to L_2 - 2L_1} \tilde{A}_1 = \begin{bmatrix} 1 & m & -1 & 1 \\ 0 & -1-2m & 3 & n-2 \\ 3 & 1 & -2 & 2n \end{bmatrix} \xrightarrow{L_3 \to L_3 - 3L_1}$$

$$\tilde{A}_2 = \begin{bmatrix} 1 & m & -1 & 1 \\ 0 & -1-2m & 3 & n-2 \\ 0 & 1-3m & 1 & 2n-3 \end{bmatrix} \xrightarrow{L_3 \to L_3 + \frac{1-3m}{1+2m} L_2} \tilde{A}_3 = \begin{bmatrix} 1 & m & -1 & 1 \\ 0 & -1-2m & 3 & n-2 \\ 0 & 0 & \frac{4-7m}{1+2m} & \frac{mn+3n-5}{1+2m} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & m & -1 \\ 0 & -1-2m & 3 \\ 0 & 0 & \frac{4-7m}{1+2m} \end{bmatrix}$$

Sistema indeterminado $\therefore$ posto A_3 = posto $\tilde{A}_3 < 3$

$\therefore \begin{cases} \frac{4-7m}{1+2m} = 0 \quad \therefore \quad 4-7m = 0 \quad \therefore \quad m = 4/7 \\ \frac{mn+3n-5}{1+2m} = 0 \quad \therefore \quad mn + 3n - 5 = 0 \quad \therefore \quad \frac{4}{7}n + 3n = 5 \end{cases}$

$\frac{25n}{7} = 5 \quad \therefore \quad n = \frac{35}{25} = \frac{7}{5}$

$\therefore \boxed{m = \frac{4}{7}, \quad n = \frac{7}{5}}$