Geometria Analítica - Prova 2

Nome:

1. A reta r

$$r: \begin{cases} x = 1 + 2t \\ y = t \\ z = 3 - t \end{cases}$$

forma um ângulo de 60^o com a reta determinada pelos pontos $A(3, 1, -2)$ e $B(4, 0, m)$. O valor de m é

(A) 2 (B) 5 (C) -4 (D) 0 (E) 3 (F) -1

2. A equação da reta perpendicular as retas r e s e passando pelo ponto de interseção das retas r e s

$$r: \begin{cases} y = -3x + 2 \\ z = 3x - 1 \end{cases} \qquad s: \frac{x}{-1} = \frac{y-1}{2} = \frac{z}{-2}$$

é dada por

$$(A) \begin{cases} x = 1 + t \\ y = -1 - t \\ z = 2 - t \end{cases} \qquad (B) \begin{cases} x = 1 \\ y + 1 = z - 2 \end{cases} \qquad (C) \begin{cases} x = 1 - t \\ y = -1 \\ z = 2 + t \end{cases}$$

$$(D) \quad x - 1 = \frac{y+1}{-1} = z - 2 \qquad (E) \begin{cases} x = t \\ y = -t \\ z = 1 + 2t \end{cases} \qquad (F) \begin{cases} y = -1 - x \\ z = 2 - x \end{cases}$$

3. A equação do plano que contém as retas de equação

$$\frac{x-4}{3} = y - 3 = \frac{z-5}{4}, \qquad \frac{x-6}{5} = \frac{y-4}{2} = \frac{z-3}{2}$$

é

(A) $6x + 4y + 3z = 12$
(B) $6x - 14y - z = -23$
(C) $4x + 3y + 5z = 13$
(D) $4x - 14y + 2z - 10 = 0$
(E) $6x - 14y - z = 0$
(F) $4x + 3y + 5z = 12$

4. $A(-1, 3, 2), B(0, 1, -1), C(-2, 0, 1), D(1, -2, 0)$ são vértices de um tetraedro. O volume é

(A) 7 (B) 30 (C) 4 (D) 23 (E) 16 (F) 5

5. Marque verdadeiro (V) ou falso (F). Cada opção marcada errada anula uma opção correta.

(V) Sejam A, B, C, D quatro pontos. Se $[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = 0$ então A, B, C, D são coplanares.
(F) $(\vec{u} \times \vec{v}) \times \vec{w} = \vec{u} \times (\vec{v} \times \vec{w}), \forall \vec{u}, \vec{v}, \vec{w}$.
(V) $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$
(F) Se $\vec{u} \times \vec{v} = \vec{0}$ então $\vec{u}$ e $\vec{v}$ são vetores nulos.

6. A equação paramétrica do plano π é

$$\pi : \begin{cases} x = 2 + t - s \\ y = -1 - t + 2s \\ z = t - s \end{cases}$$

O mesmo plano é dado pela equação geral

(A) $-x + z + 2 = 0$ (B) $y + 2z - 2 = 0$ (C) $2x - y + 3z + 9 = 0$

(D) $2x - y + 3 = 0$ (E) $x + y + 2z - 10 = 0$ (F) $x - y + z + 1 = 0$

1.

$$r: \begin{cases} x = 1+2t \\ y = t \\ z = 3-t \end{cases}$$

$\vec{u} \parallel r$: $\vec{u} = (2, 1, -1)$

$\vec{AB} = (1, -1, m+2)$

$$\cos\theta = \frac{|\vec{u} \cdot \vec{AB}|}{|\vec{u}|\,|\vec{AB}|} = \frac{|2-1-m-2|}{\sqrt{6}\,\sqrt{2+(m+2)^2}}$$

$$\cos 60^\circ = \frac{|m+1|}{\sqrt{6}\,\sqrt{2+m^2+4m+4}} = \frac{|m+1|}{\sqrt{6}\,\sqrt{m^2+4m+6}}$$

$$\frac{1}{2} = \frac{|m+1|}{\sqrt{6}\,\sqrt{m^2+4m+6}}$$

$$\frac{1}{\cancel{4}_{2}} = \frac{m^2+2m+1}{\cancel{6}_{3}(m^2+4m+6)}$$

$\therefore$ $3m^2+12m+18 = 2m^2+4m+2$

$m^2+8m+16 = 0$

$(m+4)^2 = 0$

$$\boxed{m = -4}$$

2.

$P \in r \cap s$

$$\begin{cases} y = -3x+2 \longrightarrow y = -3\cdot 1+2 = -1 \ \therefore \ \boxed{y=-1} \\ z = 3x-1 \longrightarrow 2x = 3x-1 \ \therefore \ \boxed{x=1} \\ \dfrac{x}{-1} = \dfrac{z}{-2} \ \therefore \ z = 2x \ \therefore \ \boxed{z=2} \end{cases}$$

$$\left.\begin{array}{l} \vec{u} \parallel r \ ; \ \vec{u} = (1,-3,3) \\ \vec{v} \parallel s \ ; \ \vec{v} = (-1,2,-2) \end{array}\right\} \ \vec{u}\times\vec{v} = \det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 3 \\ -1 & 2 & -2 \end{pmatrix}$$

$$\therefore \ \underbrace{\vec{u}\times\vec{v}}_{\vec{w}} = \hat{i}(6-6) - \hat{j}(-2+3) + \hat{k}(2-3)$$

$$\vec{w} = -\hat{j} - \hat{k}$$

$$/\!/ \ \vec{w} = (0,-1,-1) \ /\!/$$

ou

$$/\!/ \ \vec{w} = (0,1,1) \ /\!/$$

$$\begin{cases} x = 1 \\ y+1 = z-2 \end{cases} \qquad \begin{array}{l} x = 1 \\ y = -1 + t \\ z = 2 + t \end{array}$$

3.

$$\begin{cases} r: \dfrac{x-4}{3} = y-3 = \dfrac{z-5}{4} \\[2ex] s: \dfrac{x-6}{5} = \dfrac{y-4}{2} = \dfrac{z-3}{2} \end{cases}$$

$\vec{u} \parallel r$: $\vec{u} = (3,1,4)$

$\vec{v} \parallel s$: $\vec{v} = (5,2,2)$

$$\vec{n}_\pi = \vec{u} \times \vec{v} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 5 & 2 & 2 \end{bmatrix}$$

$$= \hat{i}(2-8) - \hat{j}(6-20) + \hat{k}(6-5)$$

$$= -6\hat{i} + 14\hat{j} + \hat{k}$$

$// \ \vec{n}_\pi = (-6, 14, 1) \ //$

$P \in r$: $P(1, b, c)$; $\dfrac{-3}{3} = b-3 = \dfrac{c-5}{4}$

$b-3 = -1 \ \therefore \ \underline{\underline{b = 2}}$

$\dfrac{c-5}{4} = -1 \ \therefore \ \underline{\underline{c}} = 5-4 = \underline{\underline{1}}$

$P(1, 2, 1)$

$\pi : -6x + 14y + z + d = 0$

$-6 + 28 + 1 + d = 0 \ \therefore \ \underline{d = -23}$

$// \ \pi : -6x + 14y + z - 23 = 0 \ //$

4. $V = \frac{1}{6}\left|[\vec{AB}, \vec{AC}, \vec{AD}]\right|$

$$\begin{cases} \vec{AB} = (1, -2, -3) \\ \vec{AC} = (-1, -3, -1) \\ \vec{AD} = (2, -5, -2) \end{cases}$$

$$\det\begin{bmatrix} 1 & -2 & -3 \\ -1 & -3 & -1 \\ 2 & -5 & -2 \end{bmatrix} = \underline{6 + 4} - 15 - 18 - 5 + \underline{4}$$

$$= \underbrace{14} - 38$$

$$= -24$$

$$V = \frac{1}{6}\left|-24\right| = 4$$

5.

(F) A, B, C, D , $[\overline{AB}, \overline{AC}, \overline{AD}] = 0 \Rightarrow A, B, C, D$ coplanares

(F) $(\vec{u} \times \vec{v}) \times \vec{w} = \bar{u} \times (\bar{v} \times \bar{w})$

(V) $\bar{u} \cdot (\bar{u} \times \bar{v}) = 0$

(F) $\bar{u} \times \bar{v} = \vec{0} \Rightarrow \bar{u} = 0 \; , \bar{v} = 0$

6.

$$\pi : \begin{cases} x = 2 + t - s \\ y = -1 - t + 2s \\ z = t - s \end{cases}$$

$$\left.\begin{array}{l} \vec{u} = (1, -1, 1) \\ \vec{v} = (-1, 2, -1) \end{array}\right\} \quad \vec{n}_\pi = \vec{u} \times \vec{v} = \det \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ -1 & 2 & -1 \end{bmatrix}$$

$$= \hat{\imath}(1 - 2) - \hat{\jmath}(-1 + 1) + \hat{k}(2 - 1)$$

$$= -\hat{\imath} + \hat{k}$$

$$= (-1, 0, 1)$$

$P(2, -1, 0) \in \pi$

$\pi : \quad -x + z + d = 0$

$P(2, -1, 0) \in \pi \quad : \quad -2 + d = 0$

$\therefore \underline{\underline{d = 2}}$

$$\therefore \boxed{\pi : \quad -x + z + 2 = 0}$$