Geometria Analítica - Prova 1

1. Seja o sistema

$$\begin{cases} x + 3y - z = 2 \\ x - y + 3z = 4 \\ 3x + y + 5z = 10 \end{cases}$$

i) Calcular o posto da matriz ampliada
(2.0) do sistema e a partir daí classifique o sistema com compatível determinado, compatível indeterminado ou incompatível

ii) Resolva o sistema. (Reduza a forma escada)
(1.0)

2. Use o método de Jordan para calcular
(2.0) a inversa da matriz

$$B = \begin{bmatrix} 1 & 3 & 2 \\ -1 & -4 & 1 \\ 2 & 6 & 5 \end{bmatrix}$$

3. Usando apenas as propriedades de determinantes
(2.0) mostre que

$$\det \begin{bmatrix} 1 & a & 2a+d \\ 1 & b & 2b+d \\ 1 & c & 2c+d \end{bmatrix} = 0$$

4. Sabendo que a matriz $S = \begin{pmatrix} 1 & x+2y & z-4 \\ 4 & 5 & 5 \\ 3z+6 & 3x-y & 0 \end{pmatrix}$
(2.0)
é simétrica determine os valores de x, y, z.

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1.

a) $\begin{cases} x+3y-z=2 \\ x-y+3z=4 \\ 3x+y+5z=10 \end{cases}$

$$A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 1 & -1 & 3 & 4 \\ 3 & 1 & 5 & 10 \end{bmatrix}$$: Matriz Ampliada

$$C = \begin{bmatrix} 1 & 3 & -1 \\ 1 & -1 & 3 \\ 3 & 1 & 5 \end{bmatrix}$$: Matriz dos coeficientes

$n = 3$: número de variáveis

Temos:

$$A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 1 & -1 & 3 & 4 \\ 3 & 1 & 5 & 10 \end{bmatrix} \xrightarrow{L_2 \to L_2 - L_1} A_1 = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -4 & 4 & 2 \\ 3 & 1 & 5 & 10 \end{bmatrix} \xrightarrow{L_3 \to L_3 - 3L_1}$$

$$A_2 = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -4 & 4 & 2 \\ 0 & -8 & 8 & 4 \end{bmatrix} \xrightarrow{L_3 \to L_3 - 2L_2} A_3 = \left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & -4 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ // Posto A = 2 // ;

Posto C = 2

$\therefore$ Posto A = Posto C < 3 $\Rightarrow$ || Sistema compatível indeterminado ||

ii)

$$A_3 = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -4 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \begin{cases} x + 3y - z = 2 \\ -4y + 4z = 2 \end{cases}$$

$$// y = z - \tfrac{1}{2} //$$

$$x = -3y + z + 2$$

$$= -3\left(z - \tfrac{1}{2}\right) + z + 2$$

$$= -3z + \tfrac{3}{2} + z + 2$$

$$// x = -2z + \tfrac{7}{2} //$$

2.

$$B = \begin{bmatrix} 1 & 3 & 2 \\ -1 & -4 & 1 \\ 2 & 6 & 5 \end{bmatrix} \;\vdots\; \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\downarrow L_2 \to L_2 + L_1$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 3 \\ 2 & 6 & 5 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\downarrow L_3 \to L_3 - 2L_1$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\downarrow L_1 \to L_1 + 3L_2$$

$$\begin{bmatrix} 1 & 0 & 11 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 4 & 3 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\downarrow L_1 \to L_1 - 11L_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 26 & 3 & -11 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\downarrow L_2 \to L_2 - 3L_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 26 & 3 & -11 \\ 7 & 1 & -3 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\downarrow L_2 \to -L_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 26 & 3 & -11 \\ -7 & -1 & 3 \\ -2 & 0 & 1 \end{bmatrix} \qquad B^{-1} = \begin{bmatrix} 26 & 3 & -11 \\ -7 & -1 & 3 \\ -2 & 0 & 1 \end{bmatrix}$$

3.

$$\det\begin{bmatrix} 1 & a & 2a+d \\ 1 & b & 2b+d \\ 1 & c & 2c+d \end{bmatrix} =$$

$$= \det\begin{bmatrix} 1 & a & 2a \\ 1 & b & 2b \\ 1 & c & 2c \end{bmatrix} + \det\begin{bmatrix} 1 & a & d \\ 1 & b & d \\ 1 & c & d \end{bmatrix}$$ 1.0

$$= 2\underbrace{\det\begin{bmatrix} 1 & a & a \\ 1 & b & b \\ 1 & c & c \end{bmatrix}}_{0} + d\underbrace{\det\begin{bmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{bmatrix}}_{=0}$$ 1.0

$= 0$

pois o det. tem duas colunas iguais

4.

$$S = \begin{pmatrix} 1 & x+2y & z-4 \\ 4 & 5 & 5 \\ 3z+6 & 3x-y & 0 \end{pmatrix}$$

S simétrica $\Rightarrow$ $S_{ij} = S_{ji}$

$$\therefore \begin{cases} x+2y = 4 \\ 3z+6 = z-4 \Rightarrow 2z = -10 \quad \therefore \underline{\underline{z=-5}} \\ 3x-y = 5 \end{cases}$$

$x+2y = 4 \Rightarrow x = 4-2y$

$3x-y = 5 \Rightarrow 3(4-2y) - y = 5$

$12 - 6y - y = 5$

$-7y = -7 \Rightarrow \underline{\underline{y=1}}$

$x = 4-2(1) = 2 \quad \therefore \underline{x=2}$

$\therefore \; // (x, y, z) = (2, 1, -5) //$