Geometria Analítica - Prova 2

1. Dado o triângulo de vértices $A(0,1,-1), B(-2,0,1)$, $C(1,-2,0)$, calcule a medida da altura relativa ao lado BC. 1,5

2. Calcular a área do paralelogramo que tem um vértice no ponto $A(3,2,1)$ e uma diagonal de extremidades $B(1,1,-1)$ e $C(0,1,2)$. Determine também a coordenada do quarto vértice do paralelogramo. 1,0

3. Determine o ângulo entre as retas r e s dadas por 1.0

$$r: \frac{x-4}{2} = \frac{y}{-1} = \frac{z+1}{-2} \qquad s: \begin{cases} x = 1 \\ \frac{y+1}{4} = \frac{z-2}{3} \end{cases}$$

4. Determine o valor de m para que as retas r e s sejam coplanares

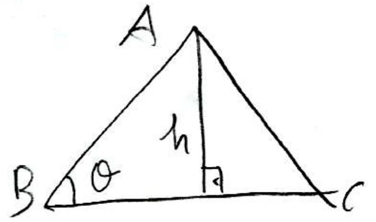
$$r: \begin{cases} x = -1 \\ y = 3 \end{cases} \qquad s: \begin{cases} y = 4x - m \\ z = x \end{cases}$$ 1.5

5. Estabelecer as equações paramétricas da reta que passa pelo ponto de interseção das retas

$$r: x - 2 = \frac{y+1}{2} = \frac{z}{3} \qquad s: \begin{cases} x = 1 - y \\ z = 2 + 2y \end{cases}$$ 2.0

e é ao mesmo tempo ortogonal a r e s.

1.



$$\begin{cases} A(0,1,-1) \\ B(-2,0,1) \\ C(1,-2,0) \end{cases}$$

$$h = |\vec{BA}| \sin\theta$$

$$\sin\theta = \frac{|\vec{BA} \times \vec{BC}|}{|\vec{BA}|\,|\vec{BC}|}$$

$$\therefore \quad h = \cancel{|\vec{BA}|} \frac{|\vec{BA} \times \vec{BC}|}{\cancel{|\vec{BA}|}\,|\vec{BC}|}$$

$$h = \frac{|\vec{BA} \times \vec{BC}|}{|\vec{BC}|}$$ 0.5

$$\vec{BA} = (2, 1, -2)$$

$$\vec{BC} = (3, -2, -1)$$

$$\vec{BA} \times \vec{BC} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -2 & -1 \end{pmatrix}$$

$$= -1\hat{i} - 6\hat{j} - 4\hat{k} - 3\hat{k} - 4\hat{i} + 2\hat{j}$$

$$= -5\hat{i} - 4\hat{j} - 7\hat{k}$$

$$\therefore \quad |\vec{BA} \times \vec{BC}| = \sqrt{25 + 16 + 49} = \sqrt{90} = 3\sqrt{10}$$

$$|\vec{BC}| = \sqrt{9+4+1} = \sqrt{14}$$

Daí

$$h = \frac{|\vec{BA} \times \vec{BC}|}{|\vec{BC}|} = \frac{3\sqrt{10}}{\sqrt{14}}$$

$$= \frac{3\cancel{\sqrt{2}}\sqrt{5}}{\cancel{\sqrt{2}}\sqrt{7}}$$

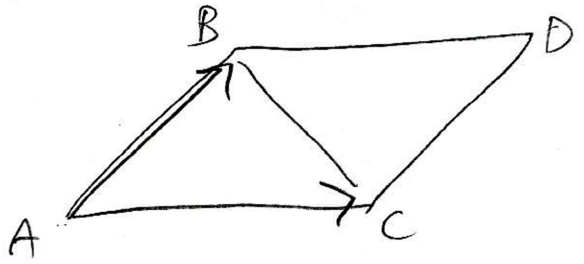
$$= \frac{3\sqrt{5}}{\sqrt{7}}$$

$$= \frac{3\sqrt{5}\sqrt{7}}{7}$$

$$= \frac{3\sqrt{35}}{7}$$

1,0

2.



$$\left\{\begin{array}{l} A(3;2;1) \\ B(1,1,-1) \\ C(0,1,2) \end{array}\right.$$

$$S_{\square} = |\vec{AB} \times \vec{AC}|$$

$$\vec{AB} = (-2,-1,-2)$$

$$\vec{AC} = (-3,-1,1)$$

$$\vec{AB} \times \vec{AC} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -1 & -2 \\ -3 & -1 & 1 \end{pmatrix}$$

$$= -\hat{i} + 6\hat{j} + 2\hat{k} + 3\hat{k} - 2\hat{i} + 2\hat{j}$$

$$= -3\hat{i} + 8\hat{j} - \hat{k}$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{9+64+1} = \sqrt{74}$$

$$\boxed{S_{\square} = \sqrt{74}} \quad 0.75$$

$D(x,y,z)$

$$\vec{CD} = (x, y-1, z-2)$$

$$\vec{CD} = \vec{AB} : \quad (x, y-1, z-2) = (-2,-1,-2)$$

$$\left.\begin{array}{l} x = -2 \\ y-1 = -1 \therefore y = 0 \\ z-2 = -2 \therefore z = 0 \end{array}\right\} \quad \underline{\underline{D(-2,0,0)}} \quad 0.25$$

3.

$$r : \frac{x-4}{2} = \frac{y}{-1} = \frac{z+1}{-2}$$

$$s : \begin{cases} x = 1 \\ \frac{y+1}{4} = \frac{z-2}{3} \end{cases}$$

$$\left.\begin{array}{ll} \vec{u} \parallel r : & \vec{u} = (2, -1, -2) \\ \vec{v} \parallel s : & \vec{v} = (0, 4, 3) \end{array}\right] \; 0.25$$

$$\cos\theta = \frac{|\vec{u} \cdot \vec{v}|}{|\vec{u}|\,|\vec{v}|}$$

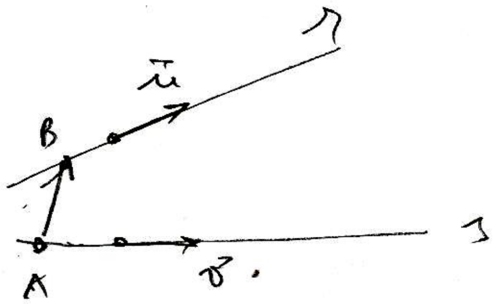
$$\left.\begin{array}{l} \vec{u} \cdot \vec{v} = (2,-1,-2) \cdot (0,4,3) \\ \quad = -4 - 6 = -10 \\ \\ |\vec{u}| = \sqrt{4+1+4} = 3 \\ |\vec{v}| = \sqrt{0+16+9} = 5 \end{array}\right\} \; 0.25$$

$$\therefore \cos\theta = \frac{|-10|}{3 \cdot 5} = \frac{\cancel{10}}{3 \cdot \cancel{5}} = \frac{2}{3}$$

$$// \; \theta = \arccos\frac{2}{3} \; // \qquad , \; 0.5$$

4. $r: \begin{cases} x=-1 \\ y=3 \end{cases}$ $\qquad s: \begin{cases} y=4x-m \\ z=x \end{cases}$

$\left.\begin{array}{l} \vec{u} \parallel r \; ; \; \vec{u} = (0,0,1) \\ \\ \vec{v} \parallel s \; ; \; \vec{v} = (1,4,1) \end{array}\right] 0.5$



$B \in r : B(-1,3,0)$

$A \in s ; A(0,-m,0)$

$\vec{AB} = (-1, 3+m, 0) \quad] \, 0.5$

Devemos ter

$[\vec{AB}, \vec{u}, \vec{v}] = 0$

$\det \begin{pmatrix} -1 & 3+m & 0 \\ 0 & 0 & 1 \\ 1 & 4 & 1 \end{pmatrix} = 0$

$3+m+4 = 0 \qquad 0.5$

$m = -7$

B.

$r : \; x-2 = \frac{y+1}{2} = \frac{z}{3}$

$s : \; \begin{cases} x = 1-y \\ z = 2+2y \end{cases}$

$$\left.\begin{array}{ll} \vec{u} \parallel r \; : & \vec{u} = (1,2,3) \\ \vec{v} \parallel s \; : & \vec{v} = (-1,1,2) \end{array}\right] 0.25$$

$$\vec{w} = \vec{u} \times \vec{v} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{pmatrix}$$

$$= 4\hat{i} - 3\hat{j} + \hat{k} + 2\hat{k} - 3\hat{i} - 2\hat{j}$$

$$= \hat{i} - 5\hat{j} + 3\hat{k}$$

0.5

$r \cap s : \; \begin{cases} x-2 = \frac{y+1}{2} = \frac{z}{3} \\ x = 1-y \\ z = 2+2y \end{cases}$

Then $\quad 1-y-2 = \frac{y+1}{2}$

$\therefore \; -y-1 = \frac{y+1}{2}, \; \therefore \; -2y-2 = y+1$

$-3y = 3 \; \therefore \; y = -1$

Daí,

$$x = 1 - y = 1 + 1 = 2 \quad \therefore \quad \underline{\underline{x = 2}}$$

$$z = 2 + 2y = 2 - 2 = 0 \quad \therefore \quad z = 0$$

$$P(2, -1, 0) \in r \cap s$$ 0,5

Queremos a reta que passa pelo ponto P e é paralela ao vetor $\vec{w}$:

$$\begin{cases} x = 2 + t \\ y = -1 - 5t \\ z = 3t \end{cases}$$ 0,5