3. $f(x) = (x-2)\sqrt{\dfrac{1+x}{1-x}}$

$$\frac{1+x}{1-x} \geq 0 \quad \text{e} \quad 1-x \neq 0$$

$- \; - \; \overset{0}{\underset{-1}{|}} \; + \; +$ $\quad 1+x$

$+ \; + \; + \; \overset{0}{\underset{1}{|}} \; - \; -$ $\quad 1-x$

$- \; \overset{0}{\underset{-1}{|}} \; + \; \overset{\nexists}{\underset{1}{|}} \; -$ $\quad \dfrac{1+x}{1-x}$

$\dfrac{1+x}{1-x} \geq 0 \Rightarrow -1 \leq x < 1$ (*)

$1-x \neq 0 \Rightarrow x \neq 1$ (**)

De (*) e (**):

$\left\{ \text{Dom} f = \{x \in \mathbb{R} : -1 \leq x < 1\} = [-1, 1) \right.$

4. $f(x) = \dfrac{1}{x + |x|}$

(*) $x + |x| = 0$.

$$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

Daí:

Se $\underline{x \geq 0}$: $x + |x| = 0$

$\therefore x + x = 0$

$2x = 0$

$\therefore \; // x = 0 //$ (compatível com a suposição $x \geq 0$)

Se $\underline{x < 0}$: $x + |x| = 0$

$\therefore x - x = 0$

$0x = 0$

$\therefore x \in \mathbb{R}$

$\therefore \; // x < 0 //$ (pois estamos tratando com $x < 0$)

Assim

$x + |x| = 0 \Rightarrow \underline{x \leq 0}$

$\therefore \; // \text{Dom } f = \{x \in \mathbb{R} : x > 0\} = (0, +\infty) //$

6. $f(x) = \sqrt{4 - \sqrt{1+9x^2}}$

$4 - \sqrt{1+9x^2} \geq 0$ e $1+9x^2 \geq 0$

$\rightarrow$ $4 - \sqrt{1+9x^2} \geq 0$

$\therefore$ $4 \geq \sqrt{1+9x^2} \geq 0$

$16 \geq 1+9x^2$

$0 \geq 9x^2 - 15$

$+ \quad - \quad +$

$-\sqrt{\frac{5}{3}} \qquad \sqrt{\frac{5}{3}}$ (*)

$\rightarrow$ $1+9x^2 \geq 0$

$\therefore$ $x \in \mathbb{R}$. (**)

De (*) e (**) tem-se $-\sqrt{\frac{5}{3}} \leq x \leq \sqrt{\frac{5}{3}}$

// Dom $f = \{x \in \mathbb{R} : -\frac{\sqrt{5}}{\sqrt{3}} \leq x \leq \sqrt{\frac{5}{3}}\} = [-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}]$ //

7. $f(x) = \frac{1}{x-1} + \frac{1}{x-2}$

$x-1 \neq 0$ e $x-2 \neq 0$

$\therefore \; x \neq 1$ e $x \neq 2$

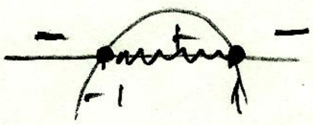
$\mathrm{Dom}\, f = \mathbb{R} - \{1,2\}$

8. $f(x) = \sqrt{1-x^2} + \sqrt{x^2-1}$

$1-x^2 \geq 0$ e $x^2-1 \geq 0$

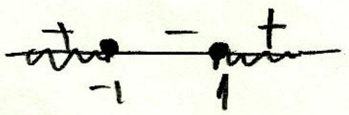
$\rightarrow \; 1-x^2 \geq 0$

$\therefore$



$(*)$

$\rightarrow \; x^2-1 \geq 0$



$(*1)$

$(*)$ e $(*1)$: $x = -1, +1$

$\therefore \; \mathrm{Dom}\, f = \{-1, 1\}$

10. $f(x) = 2x - 5 \quad , \quad x \in [-2, 2]$

$f(x) = y = 2x - 5$

$\therefore \quad x = \dfrac{y+5}{2}$

$\therefore \quad -2 \leq x \leq 2 \implies -2 \leq \dfrac{y+5}{2} \leq 2$

$\therefore -4 \leq y+5 \leq 4$

$-4-5 \leq y \leq 4-5$

$-9 \leq y \leq -1$

$\text{Im } f = [-9, -1]$

---

11. $f(x) = |x-1| \quad , \quad x \in [0, 5]$

$$f(x) = |x-1| = \begin{cases} x-1 , & x-1 \geq 0 \\ -(x-1) , & x-1 < 0 \end{cases}$$

$$\therefore \quad f(x) = \begin{cases} x-1 , & x \geq 1 \\ -x+1 , & x < 1 \end{cases}$$

Sendo $x \in [0, 5]$, consideraremos dois casos:

$\rightarrow$ Se $0 \leq x < 1$: $f(x) = -x+1$

$y = -x+1$

$\therefore x = 1-y$

$0 \leq x < 1 \Rightarrow 0 \leq 1-y < 1$

$-1 \leq -y < 0$

$\therefore 1 \geq y > 0$ (*)

$\rightarrow$ Se $1 \leq x \leq 5$: $f(x) = x-1$

$y = x-1$

$\therefore x = y+1$

$1 \leq x \leq 5 \Rightarrow 1 \leq y+1 \leq 5$

$0 \leq y \leq 4$ (*1)

De (*) e (*1) : $0 \leq y \leq 4$

// $Im\, f = [0,4]$ //

14.

$$f(x) = \frac{x^2}{x^2+1}$$

→ $\mathrm{Dom}\, f = \mathbb{R}$ ∴ $x \in \mathbb{R}$ +

→ $y = \frac{x^2}{x^2+1}$

∴ $y(x^2+1) = x^2$

$yx^2 + y = x^2$

$yx^2 - x^2 + y = 0$

$x^2(y-1) + y = 0$

$0 \le \underbrace{x^2} = \frac{-y}{y-1} = \frac{y}{1-y}$

∴ $\frac{y}{1-y} \geq 0$

		0		1	
y	$- -$	0	$+ +$		
$1-y$	$+ +$		$+$	0	$- -$
$\frac{y}{1-y}$	$-$	0	$+$	∄	$-$

∴ $0 \le y < 1$

∴ $\| \ \mathrm{Im}\, f = [0, 1) \ \|$

23.

$$f(x+1) = x^2 - 3x + 2$$

Seja $y = x+1 \therefore x = y-1$

$$\therefore \quad \underbrace{f(x+1)} = x^2 - 3x + 2$$

$$f(y) = (y-1)^2 - 3(y-1) + 2$$

$$= y^2 - 2y + 1 - 3y + 3 + 2$$

$$= y^2 - 5y + 6$$

$$\therefore \quad // f(x) = x^2 - 5x + 6 //$$

24.

$$f(x+\tfrac{1}{x}) = x^2 + \frac{1}{x^2} \qquad (x \neq 0)$$

Seja $y = x + \frac{1}{x} \Longrightarrow |y| \geq 2$ (já visto)

$$\therefore \quad y^2 = x^2 + 2 + \frac{1}{x^2}$$

$$\therefore \quad y^2 - 2 = x^2 + \frac{1}{x^2}$$

Daí $\underbrace{f(x+\tfrac{1}{x})} = \underbrace{x^2 + \frac{1}{x^2}}$

$$f(y) = y^2 - 2 \qquad \text{com } |y| \geq 2 \ (*)$$

$$\therefore \quad // f(x) = x^2 - 2 // \quad ; \quad x \in \mathbb{R} \longrightarrow$$

Obs. Note que a primeira vista, considerando (*) seríamos levados a escrever

$$f(x) = x^2 - 2 \quad \text{com} \quad |x| \geq 2$$ (onde se identificam $x \leftrightarrow y$)

Contudo isso é desnecessário.

Note que estamos pedindo a lei que defina uma função $f(x)$ de modo que se tenha

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

A função $f(x) = x^2 - 2$, $x \in \mathbb{R}$ satisfaz isso, pois,

$$f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^2 - 2$$

$$= x^2 + \cancel{2} + \frac{1}{x^2} - \cancel{2}$$

$$= x^2 + \frac{1}{x^2}$$

Assim $f(x) = x^2 - 2$, $x \in \mathbb{R}$ satisfaz a condição desejada, não sendo necessário supor $x \neq 0$ ou $|x| \geq 2$. (Na verdade, $x \neq 0$ apenas para efeito de calcular f em $x + \frac{1}{x}$, o que não restringe o domínio da função cuja lei é $f(x) = x^2 - 2$)

30. $f(x^2) = 1 - |x|^3$

Seja $y = x^2$

$\therefore \quad x = \pm\sqrt{y} \quad , \quad y \geq 0$

$\therefore \quad |x| = \sqrt{y}$

Daí $f(x^2) = 1 - |x|^3$

$\therefore \quad f(y) = 1 - \sqrt{y}^{\,3} = 1 - y^{3/2} \quad , \quad y \geq 0$

$\therefore \quad \| f(x) = 1 - x^{3/2} \quad , \quad x \geq 0 \|$