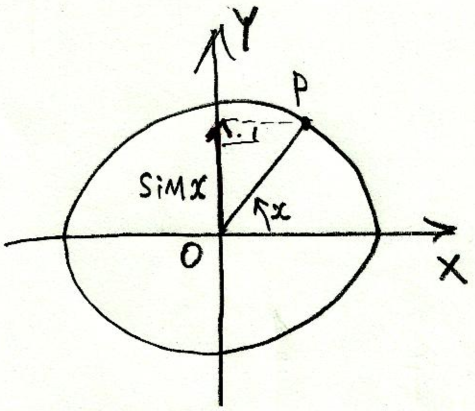
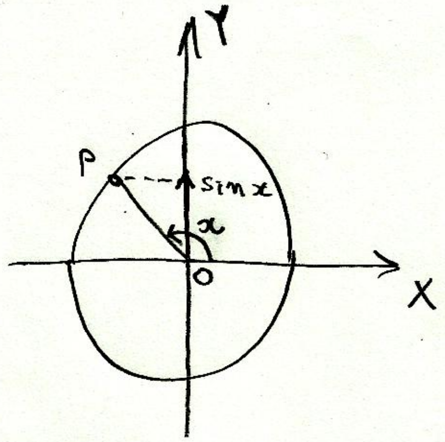
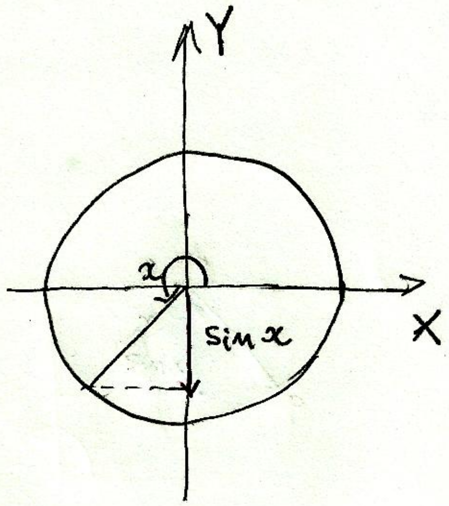
• **Def.:** Função Seno

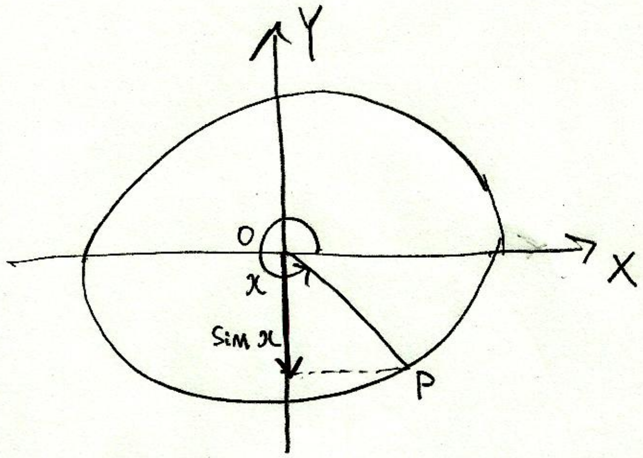
$$\sin : \mathbb{R} \to \mathbb{R}$$

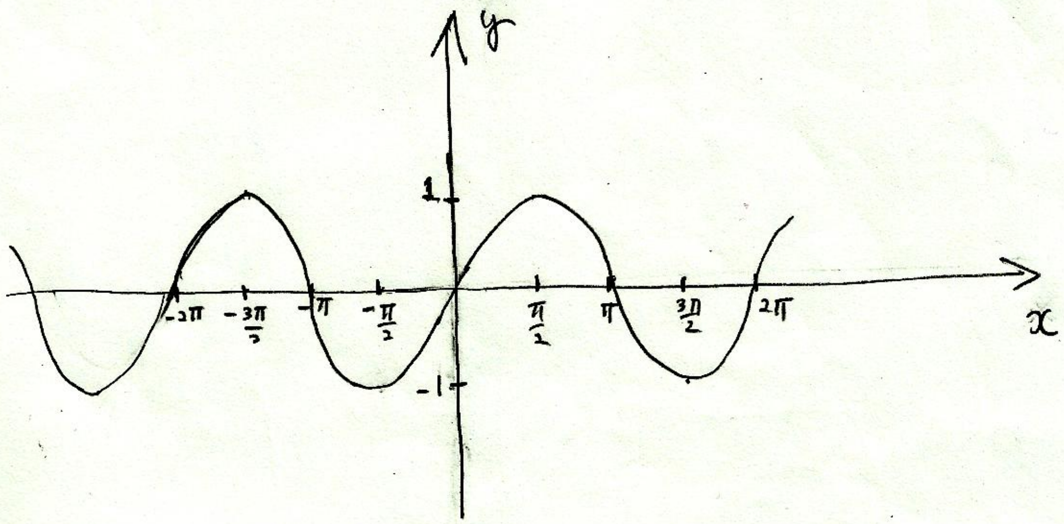
$$x \longrightarrow \sin x := \text{Projeção do segmento } \overline{OP} \text{ no eixo } OY$$





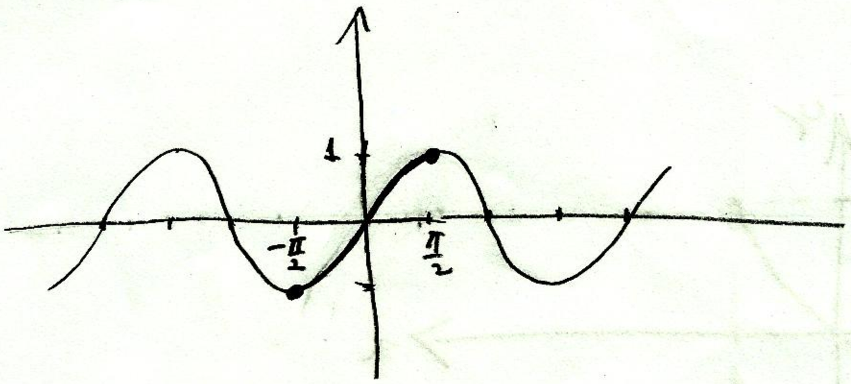






• Obs.:

Para definir a função arcoseno, inicialmente restringimos o domínio da função seno a um intervalo onde ela seja injetiva. Entre as várias possibilidades, convencionamos tomar este intervalo como sendo $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

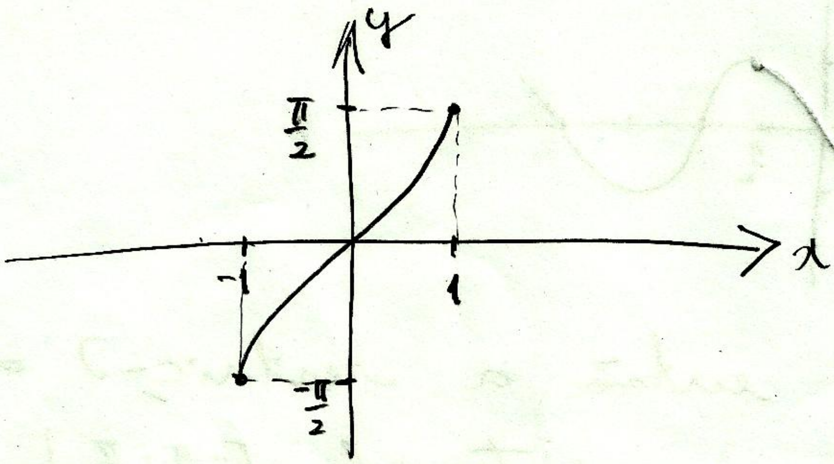


Consideramos então a restrição da função seno ao intervalo $[-\frac{\pi}{2}, \frac{\pi}{2}]$, definindo então uma função <u>bijetiva</u> denotada por $\sin\big|_{[-\frac{\pi}{2}, \frac{\pi}{2}]}$:

$$\begin{cases} \sin\big|_{[-\frac{\pi}{2}, \frac{\pi}{2}]} : [-\frac{\pi}{2}, \frac{\pi}{2}] \longrightarrow [-1, 1] \\ x \longrightarrow \sin\big|_{[-\frac{\pi}{2}, \frac{\pi}{2}]}(x) := \sin x \end{cases}$$

Definimos a função arcoseno como sendo a inversa da $\sin\big|_{[-\frac{\pi}{2}, \frac{\pi}{2}]}$, isto é

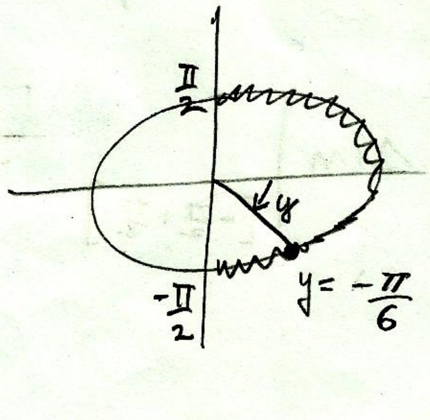
$$\begin{cases} \arcsin : [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}] \\ \qquad x \to y \doteq \arcsin x \\ \text{onde } y \text{ satisfaz } \sin y = x. \end{cases}$$



<u>Ex</u>. Calcule $\arcsin -\frac{1}{2}$.

Seja $y = \arcsin -\frac{1}{2}$

$\therefore \begin{cases} \sin y = -\frac{1}{2} \\ y \in [-\frac{\pi}{2}, \frac{\pi}{2}] \end{cases}$

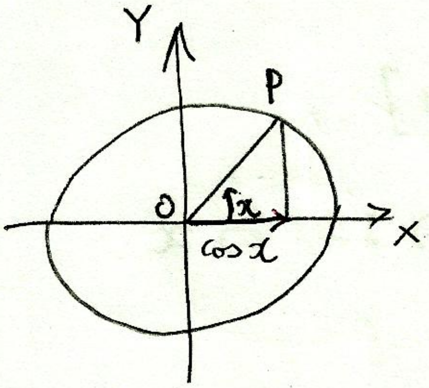


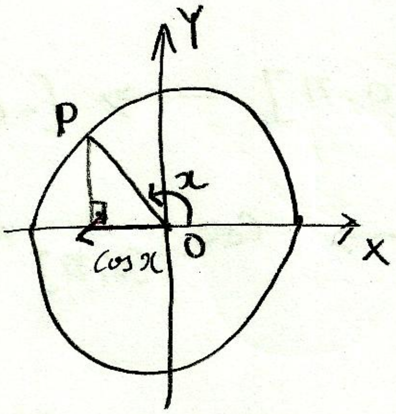
$\therefore \boxed{y = -\frac{\pi}{6}}$

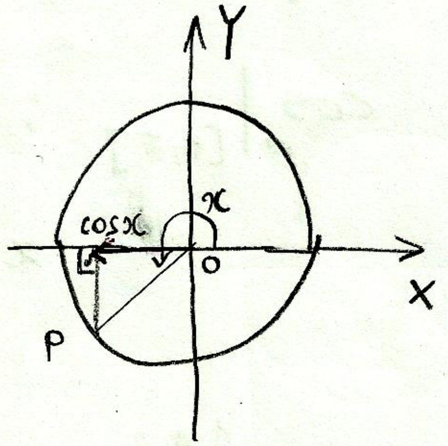
Def: <u>Função Coseno</u>

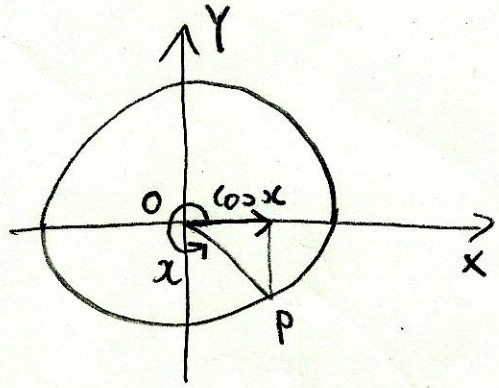
$\cos : \mathbb{R} \to \mathbb{R}$

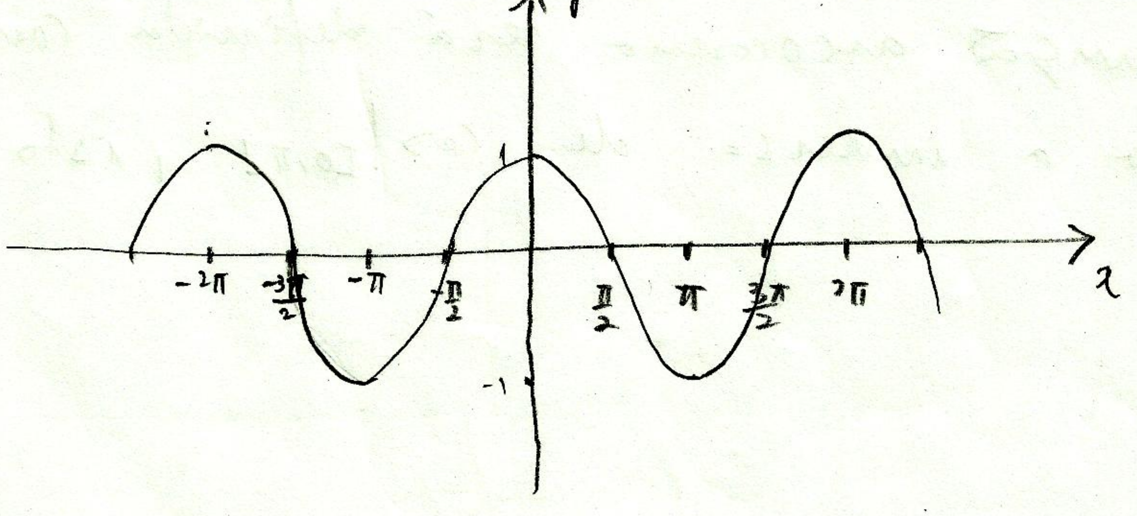
$x \to \cos x :=$ projeção do segmento $\overline{OP}$ no eixo OX





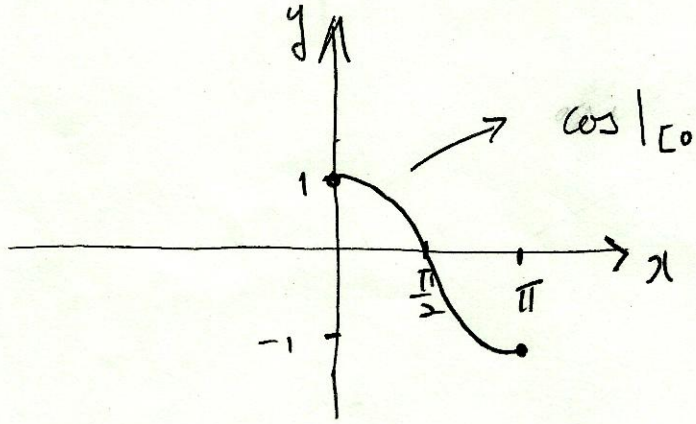






Obs.: Para definir a função arcocoseno, consideramos a função coseno restrita ao intervalo $[0,\pi]$. Definimos então a função bijetiva,

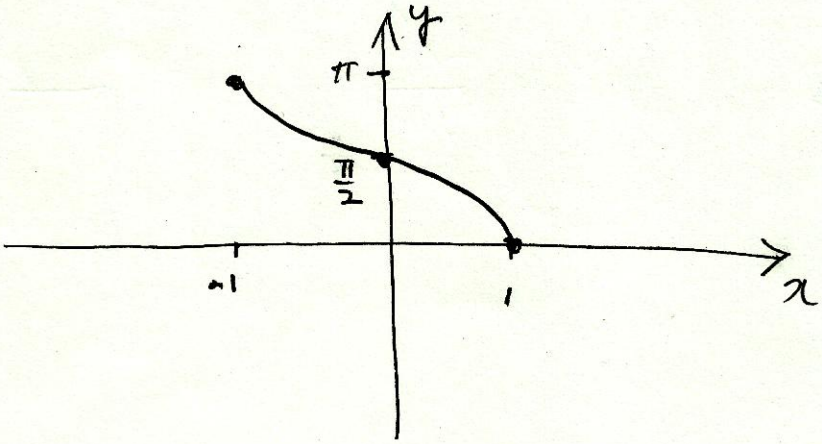
$$\cos|_{[0,\pi]} : [0,\pi] \longrightarrow [-1,1]$$
$$x \to \cos|_{[0,\pi]}(x) := \cos x$$



A função arcocoseno será definida como sendo a inversa de $\cos|_{[0,\pi]}$, isto é:

• **Def.:** Função arco coseno

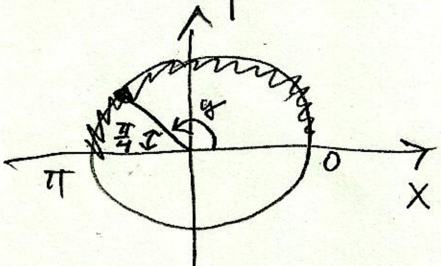
$$\begin{cases} \operatorname{arc\,cos} : [-1,1] \longrightarrow [0,\pi] \\ \qquad\qquad x \longrightarrow y \doteq \operatorname{arc\,cos} x \\ \text{onde } y \text{ satisfaz } \cos y = x \end{cases}$$



Ex. Calcule $\operatorname{arc\,cos} -\frac{\sqrt{2}}{2}$

Seja $y = \operatorname{arc\,cos} \frac{-\sqrt{2}}{2}$

$$\therefore \begin{cases} \cos y = \frac{-\sqrt{2}}{2} \\ y \in [0,\pi] \end{cases}$$



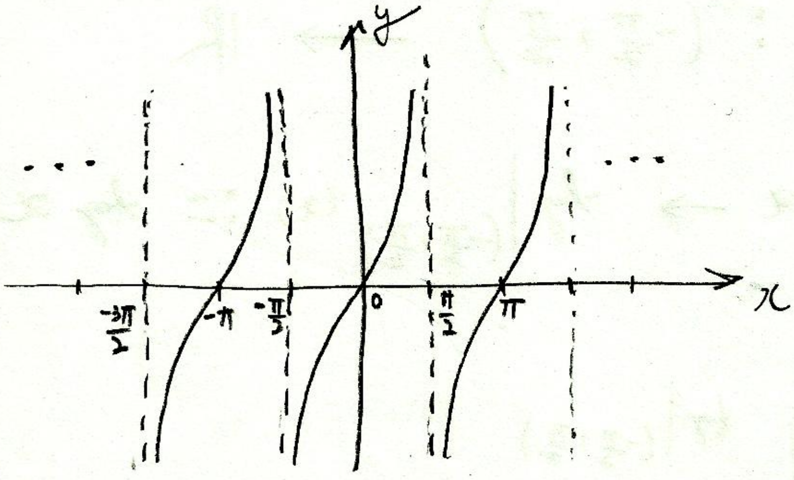
$y = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$$\therefore \quad \boxed{y = \frac{3\pi}{4}}$$

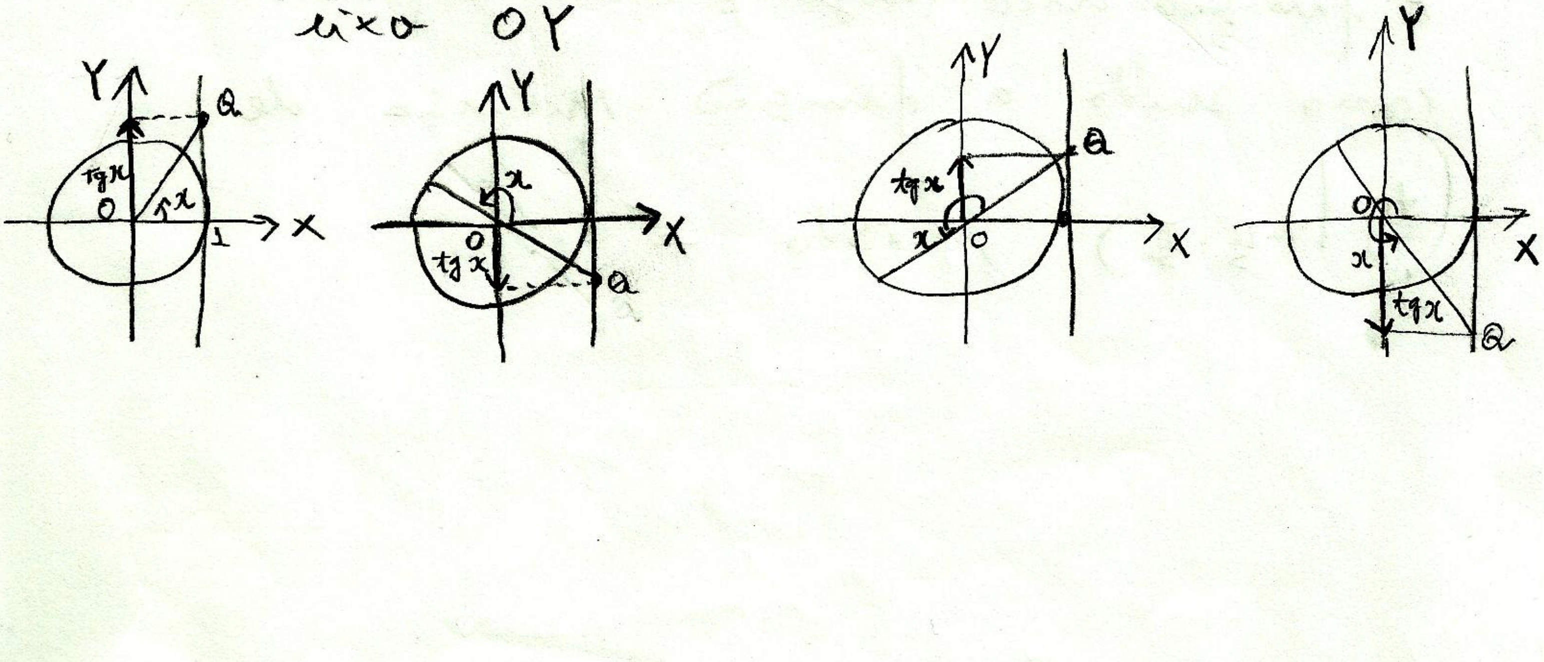
• Def. : Função tangente

$$tg : \mathbb{R} - \{(n+\tfrac{1}{2})\pi : n \in \mathbb{Z}\} \to \mathbb{R}$$

$$x \to tg\,x := \frac{\sin x}{\cos x}$$



Obs. : Em termos do ciclo trigonométrico representamos $tg\,x$ como sendo a projeção do segmento $\overline{OQ}$ no eixo OY

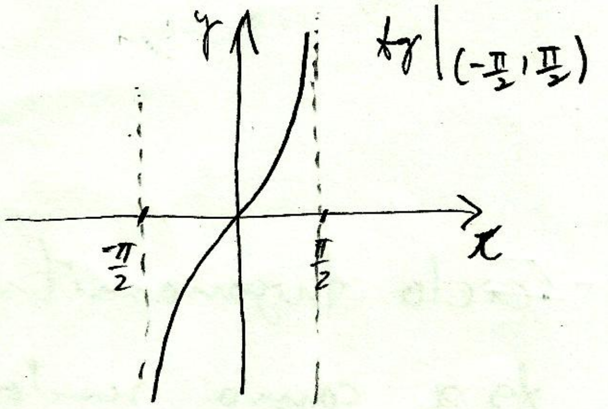


Obs.: Para definir a função arco tangente consideramos a função tangente restrita ao intervalo $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Definimos então uma função bijetiva

$$tg|_{(-\frac{\pi}{2},\frac{\pi}{2})} : (-\tfrac{\pi}{2}, \tfrac{\pi}{2}) \longrightarrow \mathbb{R}$$

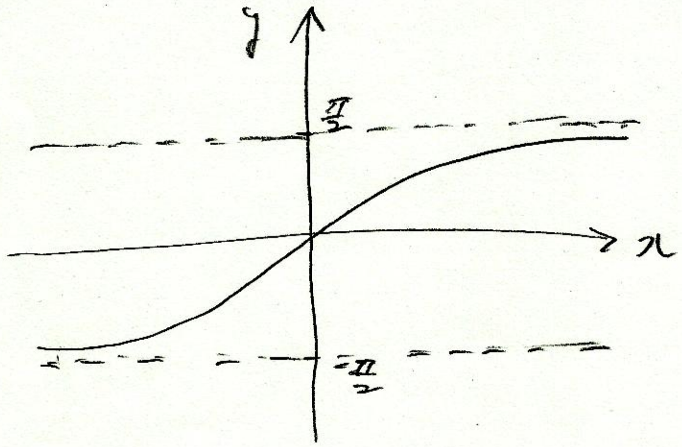
$$x \to tg|_{(-\frac{\pi}{2},\frac{\pi}{2})}(x) := tg\, x$$



A função arco tangente será definida como sendo a função inversa de $tg|_{(-\frac{\pi}{2},\frac{\pi}{2})}$, isto é:

• **Def.**: <u>Função arco tangente</u>

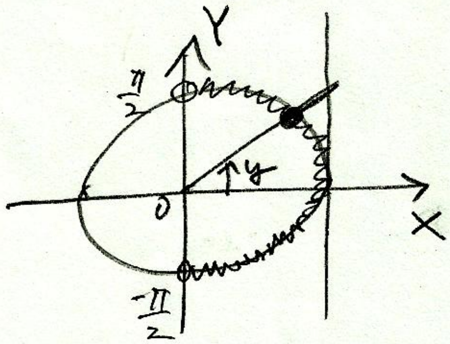
$$\begin{cases} \text{arc tg} : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \qquad x \longrightarrow y \doteq \text{arc tg}\, x \\ \text{onde } y \text{ satisfaz } \ \text{tg}\, y = x. \end{cases}$$



<u>Ex.</u> Calcule $\text{arc tg} \frac{1}{\sqrt{3}}$

Seja $y = \text{arc tg} \frac{1}{\sqrt{3}}$

$$\therefore \begin{cases} \text{tg}\, y = \frac{1}{\sqrt{3}} \\ y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \end{cases}$$



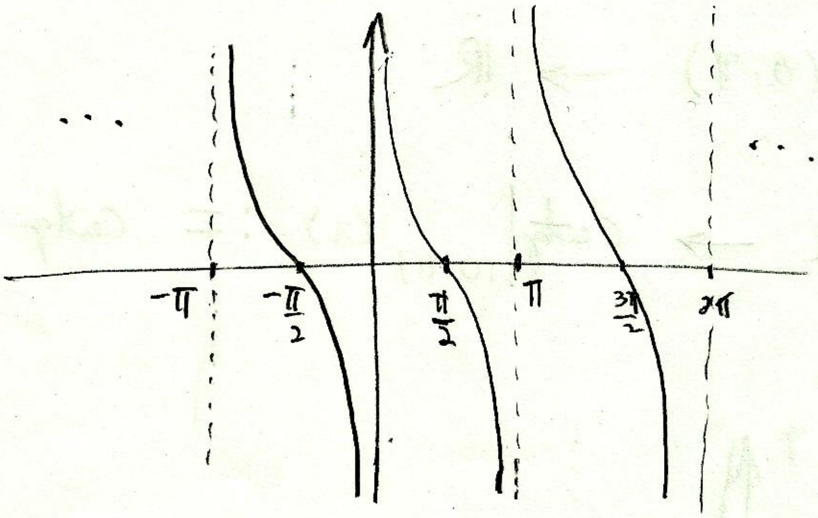
$y = \frac{\pi}{6}$

$$\therefore \boxed{y = \frac{\pi}{6}}$$

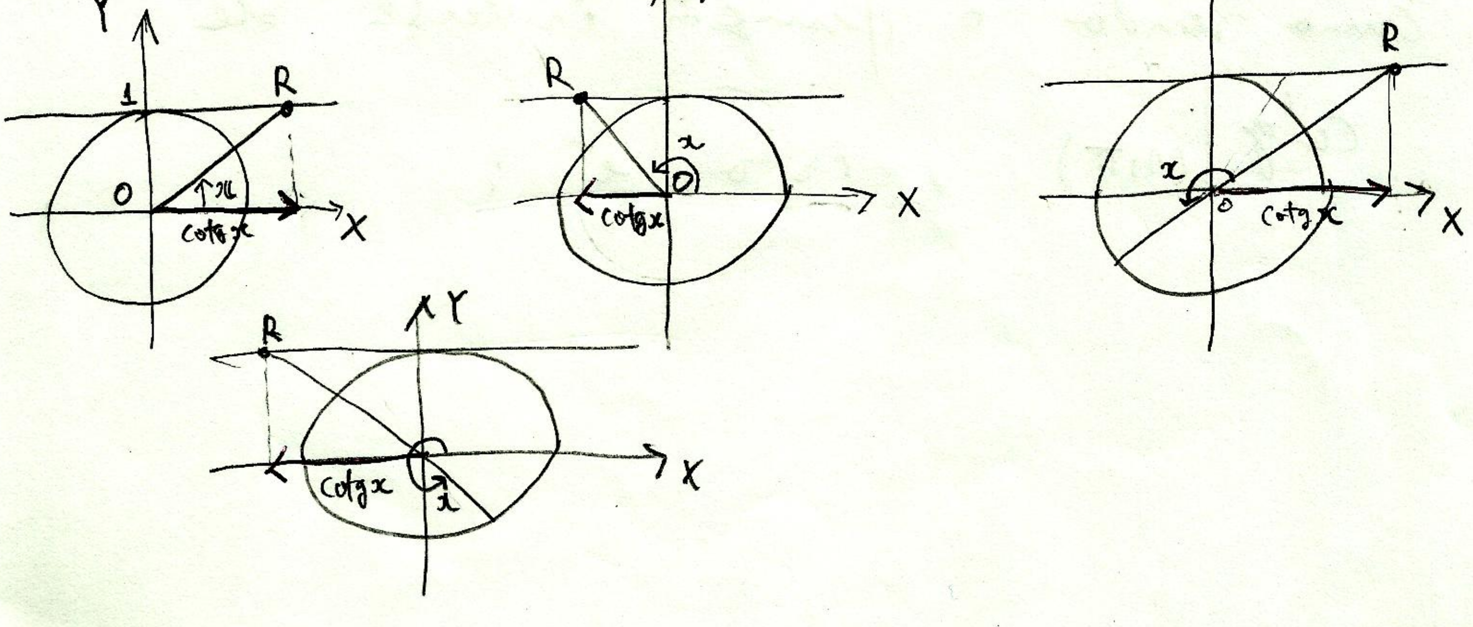
Def.: Função Cotangente

$$\text{cotg} : \mathbb{R} - \{ n\pi : n \in \mathbb{Z} \} \to \mathbb{R}$$

$$x \to \text{cotg}\, x := \frac{\cos x}{\sin x}$$



Obs.: Em termos do ciclo trigonométrico representamos cat x como sendo a projeção do segmento $\overline{OR}$ no eixo OX

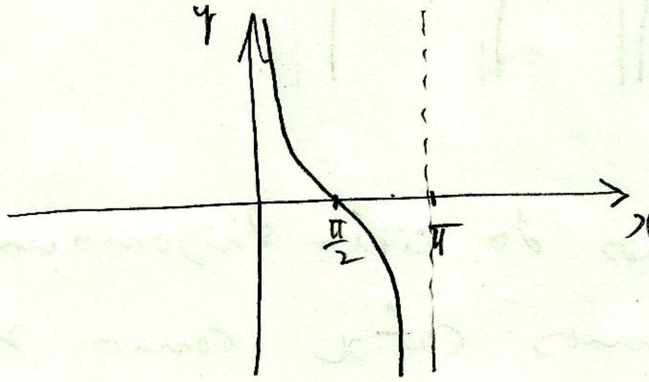


Obs.: Para definir a função arco cotangente consideremos a função cotangente restrita ao intervalo $(0,\pi)$.

Definimos então uma função bijetiva

$$\operatorname{cotg}\big|_{(0,\pi)} : (0,\pi) \to \mathbb{R}$$

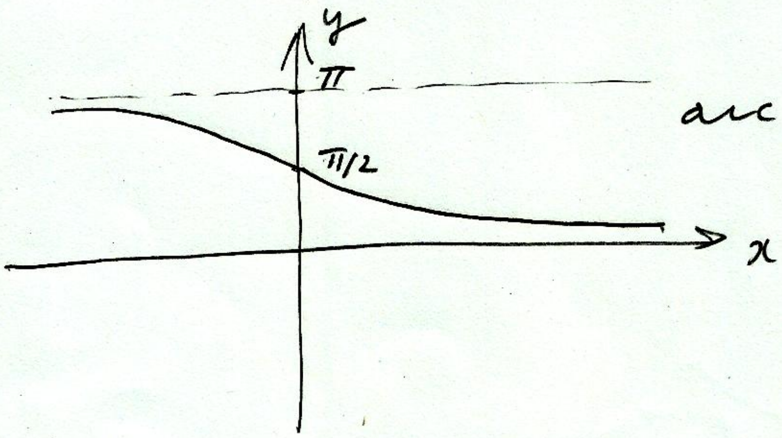
$$x \to \operatorname{cotg}\big|_{(0,\pi)}(x) := \operatorname{cotg} x$$



A função arco cotangente será definida como sendo a função inversa de $\operatorname{cotg}\big|_{(0,\pi)}$, isto é:

Def. <u>arco cotangente</u>

$$\begin{cases} \text{arc cotg} : \mathbb{R} \to (0, \pi) \\ \qquad x \to y \doteq \text{arc cotg}\, x \\ \text{onde } y \text{ satisfaz } \ \text{cotg}\, y = x \end{cases}$$

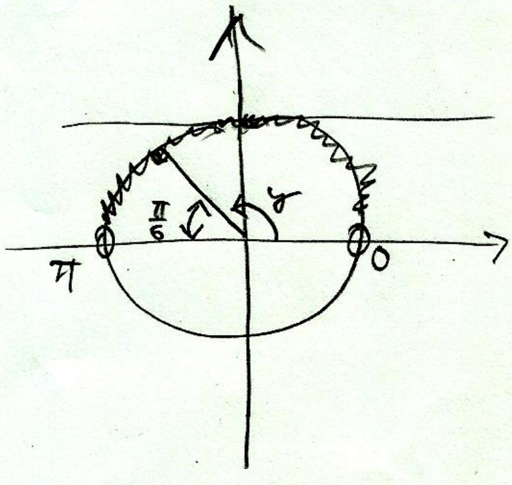


arc cotg x

<u>Ex.</u> Calcule arc cotg $-\sqrt{3}$

Seja $y = \text{arc cotg} -\sqrt{3}$

$$\therefore \begin{cases} \text{cotg}\, y = -\sqrt{3} \\ y \in (0, \pi) \end{cases}$$



$\text{cotg}\, \theta = \sqrt{3} \to \theta = \frac{\pi}{6}$

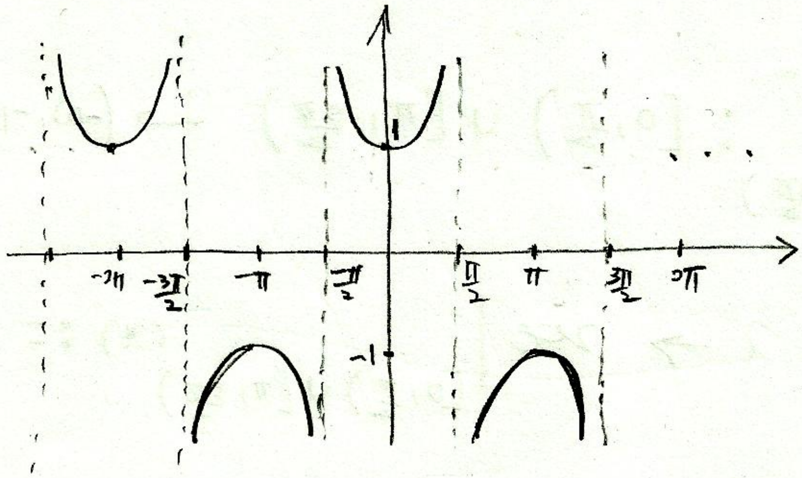
$y = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$$\therefore \boxed{y = \frac{5\pi}{6}}$$

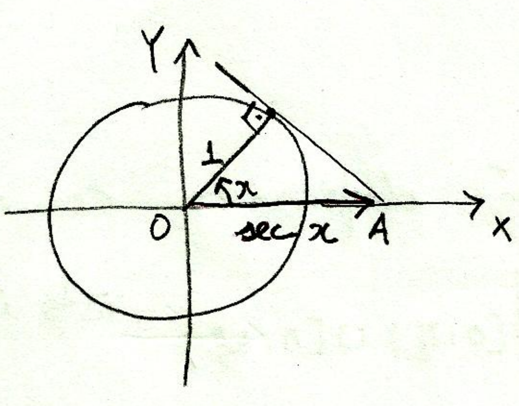
• **Def.**: Função secante

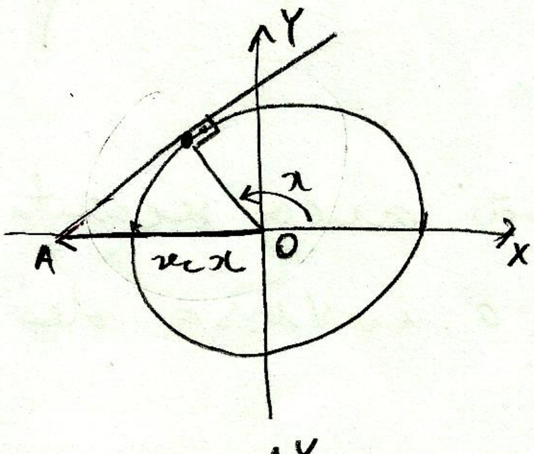
$$\sec : \mathbb{R} - \{(n+\tfrac{1}{2})\pi : n \in \mathbb{Z}\} \to \mathbb{R}$$

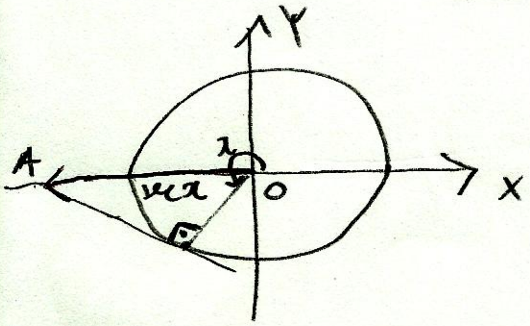
$$x \to \sec x := \frac{1}{\cos x}$$

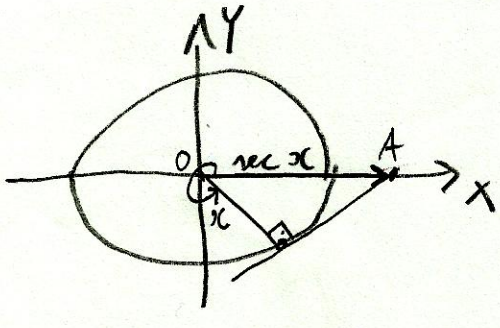


Obs.: Em termos do ciclo trigonométrico representamos $\sec x$ como sendo o comprimento do segmento $\overrightarrow{OA}$







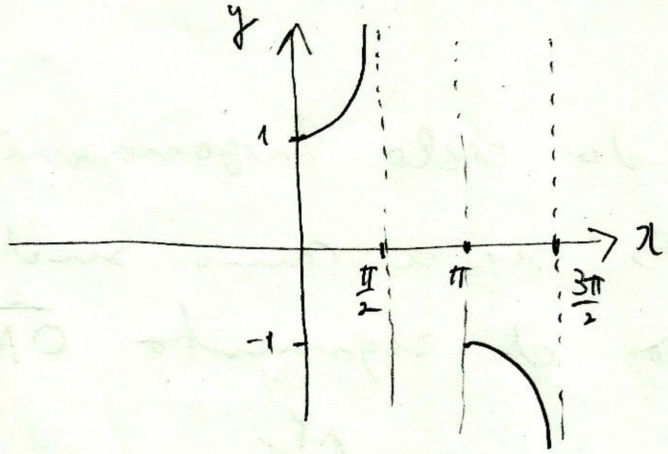


<u>Obs.</u> : Para definir a função arco secante consideremos a função secante restrita ao intervalo $[0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$

Definimos então uma função bijetiva

$$\sec\Big|_{[0,\frac{\pi}{2})\cup[\pi,\frac{3\pi}{2})} : [0,\tfrac{\pi}{2}) \cup [\pi, \tfrac{3\pi}{2}) \to (-\infty,-1] \cup [1,+\infty)$$

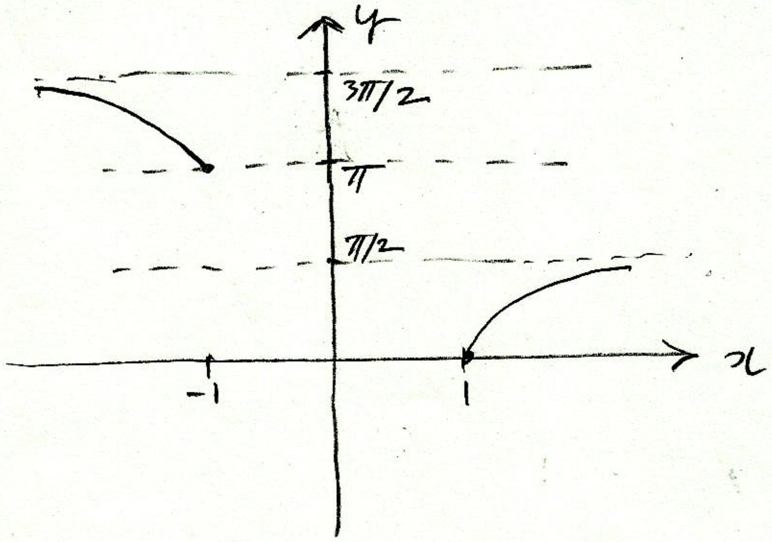
$$x \to \sec\Big|_{[0,\frac{\pi}{2})\cup[\pi,\frac{3\pi}{2})}(x) := \sec x$$



A função arco secante será definida como sendo a inversa de $\sec\Big|_{[0,\frac{\pi}{2})\cup[\pi,\frac{3\pi}{2})}$, isto é:

• Def. : arco secante

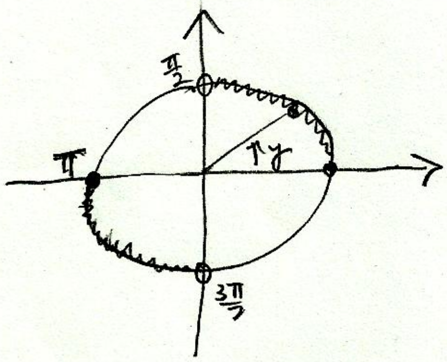
$$\begin{cases} \text{arc sec} : (-\infty,-1] \cup [1,+\infty) \longrightarrow [0,\frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2}) \\ \qquad x \to y \doteq \text{arc sec } x \\ \text{onde } y \text{ satisfaz} \quad \sec y = x \end{cases}$$



• Ex. : Calcule $\text{arc sec } \frac{2}{\sqrt{3}}$

Seja $y = \text{arc sec } \frac{2}{\sqrt{3}}$

$$\therefore \begin{cases} \sec y = \frac{2}{\sqrt{3}} \\ y \in [0,\frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2}) \quad (*) \end{cases}$$



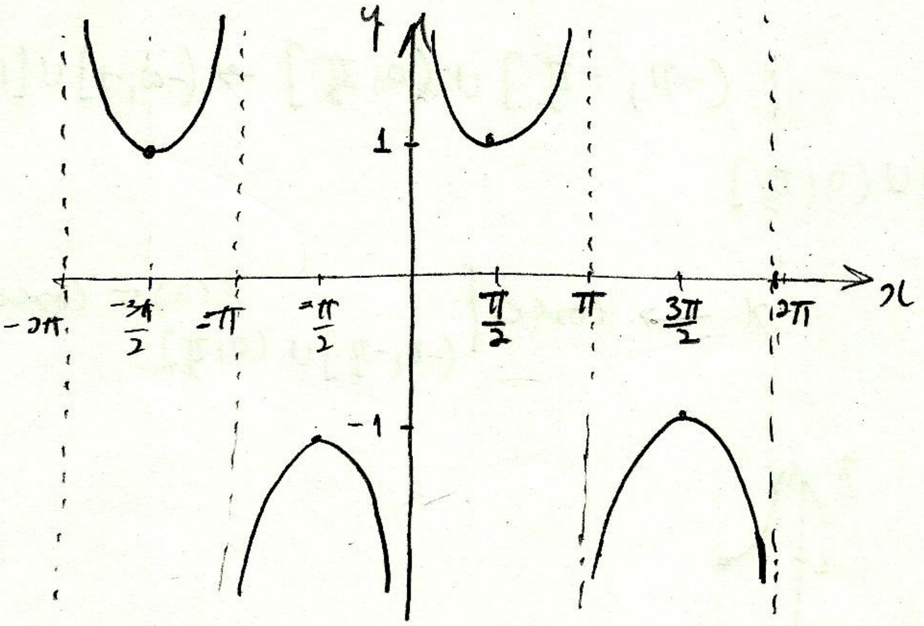
$\sec y = \frac{2}{\sqrt{3}}$ $\therefore$ $\frac{1}{\cos y} = \frac{2}{\sqrt{3}}$ $\therefore$ $\begin{cases} \cos y = \frac{\sqrt{3}}{2} \\ \therefore \\ y = \frac{\pi}{6} \text{ satisfaz } (*) \end{cases}$

$$\therefore \boxed{y = \frac{\pi}{6}}$$

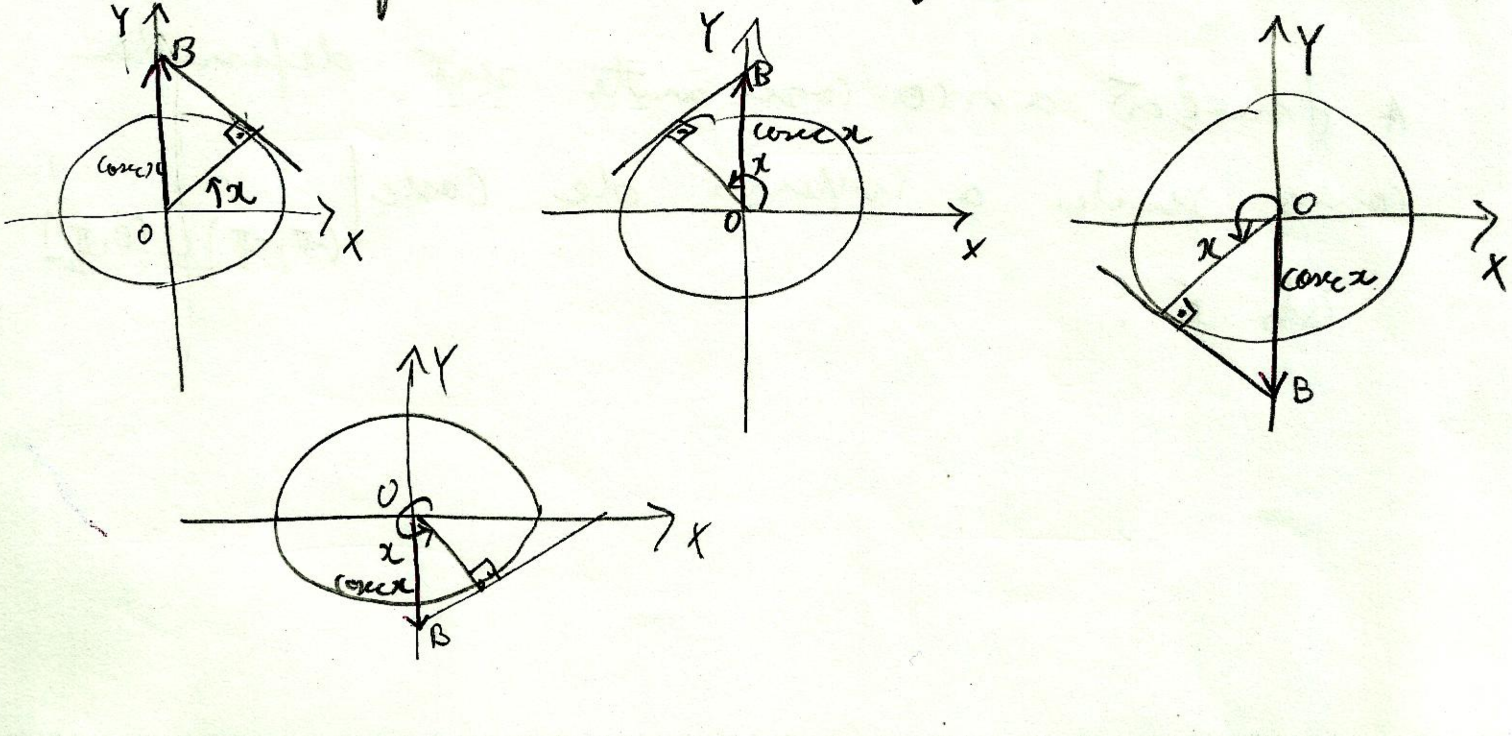
• **Def.**: Função cosecante

$$\operatorname{cosec} : \mathbb{R} - \{ n\pi : n \in \mathbb{Z} \} \to \mathbb{R}$$

$$x \longrightarrow \operatorname{cosec} x := \frac{1}{\sin x}$$



Obs.: Em termos do ciclo trigonométrico representamos cosec x como sendo o comprimento do segmento $\overline{OB}$

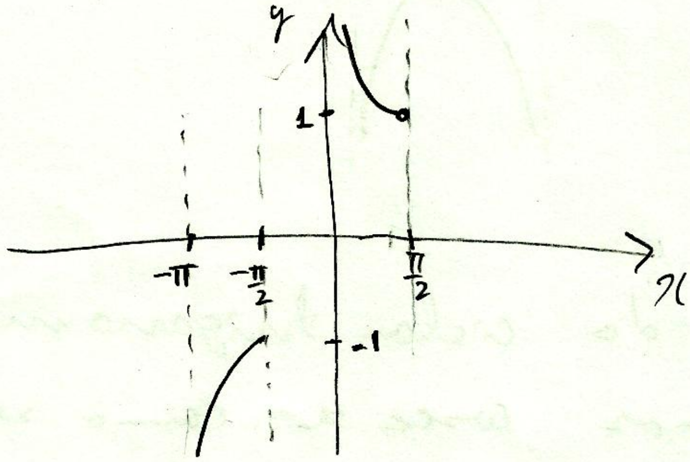


Obs.: Para definir a função arco cossecante consideramos a função cossecante restrita ao intervalo $(-\pi, -\frac{\pi}{2}] \cup (0, \frac{\pi}{2}]$.

Definimos então uma função bijetiva

$$\operatorname{cosec}\Big|_{(-\pi,-\frac{\pi}{2}]\cup(0,\frac{\pi}{2}]} : (-\pi, -\tfrac{\pi}{2}] \cup (0, \tfrac{\pi}{2}] \to (-\infty,-1] \cup [1,+\infty)$$

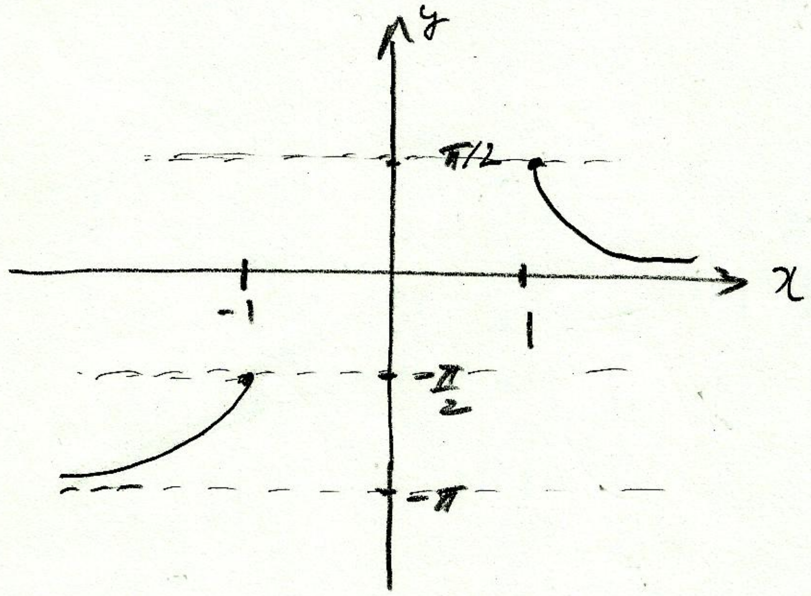
$$x \longrightarrow \operatorname{cosec}\Big|_{(-\pi,-\frac{\pi}{2}]\cup(0,\frac{\pi}{2}]}(x) := \operatorname{cosec} x$$



A função arco cossecante será definida como sendo a inversa de $\operatorname{cosec}\Big|_{(-\pi,-\frac{\pi}{2}]\cup(0,\frac{\pi}{2}]}$, isto é:

• **Def.:** arco cosecante

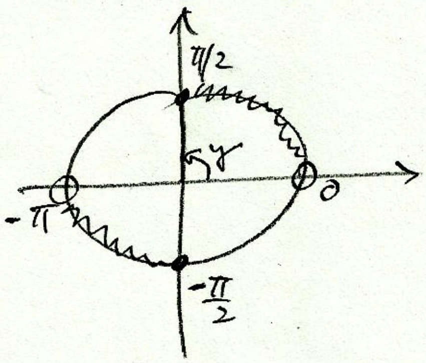
$$\begin{cases} \text{arc cosec} : (-\infty, -1] \cup [1, +\infty) \longrightarrow (-\pi, -\frac{\pi}{2}] \cup (0, \frac{\pi}{2}] \\ x \longrightarrow y \doteq \text{arc cosec } x \\ \text{onde } y \text{ satisfaz } \operatorname{cosec} y = x \end{cases}$$



Ex. Calcule arc cosec 1

Seja $y = \text{arc cosec } 1$

$$\therefore \begin{cases} \operatorname{cosec} y = 1 \\ y \in (-\pi, -\frac{\pi}{2}] \cup (0, \frac{\pi}{2}] \end{cases}$$



$\operatorname{cosec} y = 1$

$\therefore \dfrac{1}{\sin y} = 1 \quad \therefore \sin y = 1 \quad \therefore \boxed{y = \frac{\pi}{2}}$