6.1 EXERCISES

1–4 Find the area of the shaded region.

5–28 Sketch the region enclosed by the given curves. Decide whether to integrate with respect to $x$ or $y$. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

5. $y = x + 1$, $y = 9 - x^2$, $x = -1$, $x = 2$
6. $y = \sin x$, $y = e^x$, $x = 0$, $x = \pi/2$
7. $y = x$, $y = x^2$
8. $y = x^2 - 2x$, $y = x + 4$
9. $y = 1/x$, $y = 1/x^2$, $x = 2$
10. $y = 1 + \sqrt{x}$, $y = (3 + x)/3$
11. $y = x^2$, $y^2 = x$
12. $y = x^3$, $y = 4x - x^3$
13. $y = 12 - x^2$, $y = x^2 - 6$
14. $y = \cos x$, $y = 2 - \cos x$, $0 \leq x \leq 2\pi$
15. $y = \tan x$, $y = 2 \sin x$, $-\pi/3 \leq x \leq \pi/3$
16. $y = x^3 - x$, $y = 3x$
17. $y = \sqrt{x}$, $y = \frac{1}{2}x$, $x = 9$
18. $y = 8 - x^2$, $y = x^2$, $x = -3$, $x = 3$
19. $x = 2y^2$, $x = 4 + y^2$
20. $4x + y^2 = 12$, $x = y$

21. $x = 1 - y^2$, $x = y^2 - 1$
22. $y = \sin(\pi x/2)$, $y = x$
23. $y = \cos x$, $y = \sin 2x$, $x = 0$, $x = \pi/2$
24. $y = \cos x$, $y = 1 - \cos x$, $0 \leq x \leq \pi$
25. $y = x^2$, $y = 2/(x^2 + 1)$
26. $y = |x|$, $y = x^2 - 2$
27. $y = 1/x$, $y = x$, $y = \sqrt[4]{x}$, $x > 0$
28. $y = 3x^2$, $y = 8x^3$, $4x + y = 4$, $x \geq 0$

29–30 Use calculus to find the area of the triangle with the given vertices.
29. $(0, 0)$, $(2, 1)$, $(-1, 6)$
30. $(0, 5)$, $(2, -2)$, $(5, 1)$

31–32 Evaluate the integral and interpret it as the area of a region. Sketch the region.
31. $\int_0^{e^2} |\sin x - \cos 2x| \, dx$
32. $\int_0^4 \sqrt{x + 2} - x \, dx$

33–34 Use the Midpoint Rule with $n = 4$ to approximate the area of the region bounded by the given curves.
33. $y = \sin^2(\pi x/4)$, $y = \cos^2(\pi x/4)$, $0 \leq x \leq 1$
34. $y = \sqrt{16 - x^2}$, $y = x$, $x = 0$

35–38 Use a graph to find approximate $x$-coordinates of the points of intersection of the given curves. Then find (approximately) the area of the region bounded by the curves.
35. $y = x \sin(x^2)$, $y = x^4$
36. $y = e^x$, $y = 2 - x^2$
37. $y = 3x^2 - 2x$, $y = x^3 - 3x + 4$
38. $y = x \cos x$, $y = x^{10}$
39. Use a computer algebra system to find the exact area enclosed by the curves \( y = x^2 - 6x^3 + 4x \) and \( y = x \).

40. Sketch the region in the xy-plane defined by the inequalities \( x - 2y^2 \geq 0 \), \( 1 - x - |y| \geq 0 \) and find its area.

41. Racing cars driven by Chris and Kelly are side by side at the start of a race. The figure shows the graphs of their velocity functions. Two cars, A and B, start side by side and accelerate from rest.

(a) Which car is ahead after one minute? Explain.
(b) What is the meaning of the area of the shaded region?
(c) Which car is ahead after two minutes? Explain.
(d) Estimate the time at which the cars are again side by side.

\[
\begin{array}{c|c|c|c|c|c|c}
 t & v_C & v_K & t & v_C & v_K \\
0 & 0 & 0 & 6 & 69 & 80 \\
1 & 20 & 22 & 7 & 75 & 86 \\
2 & 32 & 37 & 8 & 81 & 93 \\
3 & 46 & 52 & 9 & 86 & 98 \\
4 & 54 & 61 & 10 & 90 & 102 \\
5 & 62 & 71 & & & \\
\end{array}
\]

42. The widths (in meters) of a kidney-shaped swimming pool were measured at 2-meter intervals as indicated in the figure. Use the Midpoint Rule to estimate the area of the pool.

43. A cross-section of an airplane wing is shown. Measurements of the height of the wing, in centimeters, at 20-centimeter intervals are 5.8, 20.3, 26.7, 29.0, 27.6, 27.3, 23.8, 20.5, 15.1, 8.7, and 2.8. Use the Midpoint Rule to estimate the area of the wing’s cross-section.

44. If the birth rate of a population is \( b(t) = 2200e^{0.02t} \) people per year and the death rate is \( d(t) = 1460e^{0.013t} \) people per year, find the area between these curves for \( 0 \leq t \leq 10 \). What does this area represent?

45. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.
(a) Which car is ahead after one minute? Explain.
(b) What is the meaning of the area of the shaded region?
(c) Which car is ahead after two minutes? Explain.
(d) Estimate the time at which the cars are again side by side.

46. The figure shows graphs of the marginal revenue function \( R' \) and the marginal cost function \( C' \) for a manufacturer. [Recall from Section 4.7 that \( R(x) \) and \( C(x) \) represent the revenue and cost when \( x \) units are manufactured. Assume that \( R \) and \( C \) are measured in thousands of dollars.] What is the meaning of the area of the shaded region? Use the Midpoint Rule to estimate the value of this quantity.

47. The curve with equation \( y^2 = x^3(x + 3) \) is called Tschirnhausen’s cubic. If you graph this curve you will see that part of the curve forms a loop. Find the area enclosed by the loop.

48. Find the area of the region bounded by the parabola \( y = x^2 \), the tangent line to this parabola at \((1, 1)\), and the x-axis.

49. Find the number \( b \) such that the line \( y = b \) divides the region bounded by the curves \( y = x^2 \) and \( y = 4 \) into two regions with equal area.

50. (a) Find the number \( a \) such that the line \( x = a \) bisects the area under the curve \( y = 1/x^2 \), \( 1 \leq x \leq 4 \).
(b) Find the number \( b \) such that the line \( y = b \) bisects the area in part (a).

51. Find the values of \( c \) such that the area of the region bounded by the parabolas \( y = x^2 - c^2 \) and \( y = c^2 - x^2 \) is 576.

52. Suppose that \( 0 < c < \pi/2 \). For what value of \( c \) is the area of the region enclosed by the curves \( y = \cos x \), \( y = \cos(x - c) \), and \( x = 0 \) equal to the area of the region enclosed by the curves \( y = \cos(x - c) \), \( x = \pi \), and \( y = 0 \)?

53. For what values of \( m \) do the line \( y = mx \) and the curve \( y = x/(x^2 + 1) \) enclose a region? Find the area of the region.
we would have obtained the integral

\[ V = \int_0^h \frac{L^2}{h^2} (h - y)^3 \, dy = \frac{L^2 h}{3} \]

**EXAMPLE 9** A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of 30° along a diameter of the cylinder. Find the volume of the wedge.

**SOLUTION** If we place the \(x\)-axis along the diameter where the planes meet, then the base of the solid is a semicircle with equation \(y = \sqrt{16 - x^2}, -4 \leq x \leq 4\). A cross-section perpendicular to the \(x\)-axis at a distance \(x\) from the origin is a triangle \(ABC\), as shown in Figure 17, whose base is \(y = \sqrt{16 - x^2}\) and whose height is \(|BC| = y \tan 30^\circ = \frac{\sqrt{16 - x^2}}{\sqrt{3}}\). Thus the cross-sectional area is

\[ A(x) = \frac{1}{2} \sqrt{16 - x^2} \cdot \frac{1}{\sqrt{3}} \sqrt{16 - x^2} = \frac{16 - x^2}{2\sqrt{3}} \]

and the volume is

\[ V = \int_{-4}^{4} A(x) \, dx = \int_{-4}^{4} \frac{16 - x^2}{2\sqrt{3}} \, dx = \frac{1}{\sqrt{3}} \int_{0}^{4} (16 - x^2) \, dx = \frac{1}{\sqrt{3}} \left[ 16x - \frac{x^3}{3} \right]_0^4 = \frac{128}{3\sqrt{3}} \]

**FIGURE 17**

For another method see Exercise 64.

### 6.2 EXERCISES

1–18. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

1. \(y = 2 - \frac{1}{4} x, \quad y = 0, \quad x = 1, \quad x = 2; \quad \text{about the } x\)-axis
2. \(y = 1 - x^2, \quad y = 0; \quad \text{about the } x\)-axis
3. \(y = 1/x, \quad x = 1, \quad x = 2, \quad y = 0; \quad \text{about the } x\)-axis
4. \(y = \sqrt{25 - x^2}, \quad y = 0, \quad x = 2, \quad x = 4; \quad \text{about the } x\)-axis
5. \(x = 2y, \quad x = 0, \quad y = 9; \quad \text{about the } y\)-axis
6. \(y = \ln x, \quad y = 1, \quad y = 2, \quad x = 0; \quad \text{about the } y\)-axis
7. \(y = x^3, \quad y = x, \quad x \geq 0; \quad \text{about the } x\)-axis
8. \(y = \frac{1}{4} x^2, \quad y = 5 - x^2; \quad \text{about the } x\)-axis
9. \(y^2 = x, \quad x = 2y; \quad \text{about the } y\)-axis
10. \(y = \frac{1}{4} x^2, \quad x = 2, \quad y = 0; \quad \text{about the } y\)-axis
11. \(y = x, \quad y = \sqrt{x}; \quad \text{about } y = 1\)
12. \(y = e^{-x}, \quad y = 1, \quad x = 2; \quad \text{about } y = 2\)
13. \(y = 1 + \sec x, \quad y = 3; \quad \text{about } y = 1\)
14. \(y = 1/x, \quad y = 0, \quad x = 1, \quad x = 3; \quad \text{about } y = -1\)
15. \(x = y^2, \quad x = 1; \quad \text{about } x = 1\)
16. \(y = x, \quad y = \sqrt{x}; \quad \text{about } x = 2\)
17. \(y = x^2, \quad x = y^2; \quad \text{about } x = -1\)
18. \(y = x, \quad y = 0, \quad x = 2, \quad x = 4; \quad \text{about } x = 1\)
19–30 Refer to the figure and find the volume generated by rotating the given region about the specified line.

![Figure showing region and lines](image)

<table>
<thead>
<tr>
<th>Region</th>
<th>Axis of Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{R}_1 )</td>
<td>( OA )</td>
</tr>
<tr>
<td>( \mathcal{R}_2 )</td>
<td>( AB )</td>
</tr>
<tr>
<td>( \mathcal{R}_2 )</td>
<td>( OA )</td>
</tr>
<tr>
<td>( \mathcal{R}_3 )</td>
<td>( AB )</td>
</tr>
<tr>
<td>( \mathcal{R}_3 )</td>
<td>( AB )</td>
</tr>
</tbody>
</table>

31–36 Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

31. \( y = \tan^3 x \), \( y = 1 \), \( x = 0 \); about \( y = 1 \)
32. \( y = (x - 2)^4 \), \( 8x - y = 16 \); about \( x = 10 \)
33. \( y = 0 \), \( y = \sin x \), \( 0 \leq x \leq \pi \); about \( y = 1 \)
34. \( y = 0 \), \( y = \sin x \), \( 0 \leq x \leq \pi \); about \( y = -2 \)
35. \( x^2 - y^2 = 1 \), \( x = 3 \); about \( x = -2 \)
36. \( y = \cos x \), \( y = 2 - \cos x \), \( 0 \leq x \leq 2\pi \); about \( y = 4 \)

37–38 Use a graph to find approximate \( x \)-coordinates of the points of intersection of the given curves. Then use your calculator to find (approximately) the volume of the solid obtained by rotating about the \( x \)-axis the region bounded by these curves.

37. \( y = 2 + x^2 \cos x \), \( y = x^4 + x + 1 \)
38. \( y = 3 \sin(x^2) \), \( y = e^{x/2} + e^{-2x} \)

39–40 Use a computer algebra system to find the exact volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

39. \( y = \sin^2 x \), \( y = 0 \), \( 0 \leq x \leq \pi \); about \( y = -1 \)
40. \( y = x \), \( y = x e^{1-x^2} \); about \( y = 3 \)

41–44 Each integral represents the volume of a solid. Describe the solid.

41. \( \pi \int_0^{\pi/2} \cos^2 x \, dx \)
42. \( \pi \int_2^{\infty} y \, dy \)
43. \( \pi \int_0^{\pi/2} (y^4 - y^8) \, dy \)
44. \( \pi \int_0^{\pi/2} [(1 + \cos x)^2 - 1^2] \, dx \)

45. A CAT scan produces equally spaced cross-sectional views of a human organ that provide information about the organ otherwise obtained only by surgery. Suppose that a CAT scan of a human liver shows cross-sections spaced 1.5 cm apart. The liver is 15 cm long and the cross-sectional areas, in square centimeters, are 0, 18, 58, 79, 94, 106, 117, 128, 63, 39, and 0. Use the Midpoint Rule to estimate the volume of the liver.

46. A log 10 m long is cut at 1-meter intervals and its cross-sectional areas \( A \) (at a distance \( x \) from the end of the log) are listed in the table. Use the Midpoint Rule with \( n = 5 \) to estimate the volume of the log.

<table>
<thead>
<tr>
<th>( x ) (m)</th>
<th>( A ) (m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.68</td>
</tr>
<tr>
<td>1</td>
<td>0.65</td>
</tr>
<tr>
<td>2</td>
<td>0.64</td>
</tr>
<tr>
<td>3</td>
<td>0.61</td>
</tr>
<tr>
<td>4</td>
<td>0.58</td>
</tr>
<tr>
<td>5</td>
<td>0.59</td>
</tr>
</tbody>
</table>

47. (a) If the region shown in the figure is rotated about the \( x \)-axis to form a solid, use the Midpoint Rule with \( n = 4 \) to estimate the volume of the solid.

(b) Estimate the volume if the region is rotated about the \( y \)-axis. Again use the Midpoint Rule with \( n = 4 \).

48. (a) A model for the shape of a bird’s egg is obtained by rotating about the \( x \)-axis the region under the graph of

\[
f(x) = (ax^3 + bx^2 + cx + d)\sqrt{1 - x^2}
\]

Use a CAS to find the volume of such an egg.
(b) For a Red-throated Loon, \( a = -0.06 \), \( b = 0.04 \), \( c = 0.1 \), and \( d = 0.54 \). Graph \( f \) and find the volume of an egg of this species.

49–61 Find the volume of the described solid \( S \).

49. A right circular cone with height \( h \) and base radius \( r \)

50. A frustum of a right circular cone with height \( h \), lower base radius \( R \), and top radius \( r \)
8.1 Exercises

1. Use the arc length formula (3) to find the length of the curve \( y = 2x - 5, -1 \leq x \leq 3 \). Check your answer by noting that the curve is a line segment and calculating its length by the distance formula.

2. Use the arc length formula to find the length of the curve \( y = \sqrt{2 - x^2}, 0 \leq x \leq 1 \). Check your answer by noting that the curve is part of a circle.

3–6 Set up, but do not evaluate, an integral for the length of the curve.

3. \( y = \cos x, \quad 0 \leq x \leq 2\pi \)

4. \( y = xe^{-x^2}, \quad 0 \leq x \leq 1 \)

5. \( x = y + y^3, \quad 1 \leq y \leq 4 \)

6. \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \)

7–12 Find the length of the curve.

7. \( y = 1 + 6x^{3/2}, \quad 0 \leq x \leq 1 \)

8. \( y^2 = 4(x + 4)^2, \quad 0 \leq x \leq 2, \quad y > 0 \)

9. \( y = \frac{x^5}{6} + \frac{1}{10x^3}, \quad 1 \leq x \leq 2 \)

10. \( x = \frac{y^4}{8} + \frac{1}{4y^2}, \quad 1 \leq y \leq 2 \)

11. \( x = \frac{1}{2} \sqrt{y} (y - 3), \quad 1 \leq y \leq 9 \)

12. \( y = \ln(\cos x), \quad 0 \leq x \leq \pi/3 \)

13. \( y = \ln(\sec x), \quad 0 \leq x \leq \pi/4 \)

14. \( y = 3 + \frac{1}{2} \cosh 2x, \quad 0 \leq x \leq 1 \)

15. \( y = \ln(1 - x^2), \quad 0 \leq x \leq \frac{1}{2} \)

16. \( y = \sqrt{x - x^2} + \sin^{-1}(\sqrt{x}) \)

17. \( y = e^x, \quad 0 \leq x \leq 1 \)

18. \( y = \ln\left(\frac{e^x + 1}{e^x - 1}\right), \quad a \leq x \leq b, \quad a > 0 \)

19–20 Find the length of the arc of the curve from point \( P \) to point \( Q \).

19. \( y = \frac{1}{7} x^2, \quad P(-1, \frac{1}{7}), \quad Q(1, \frac{1}{7}) \)

20. \( x^2 = (y - 4)^2, \quad P(1, 5), \quad Q(8, 8) \)

21–22 Graph the curve and visually estimate its length. Then find its exact length.

21. \( y = \frac{1}{3} (x^2 - 1)^{3/2}, \quad 1 \leq x \leq 3 \)

22. \( y = \frac{x^3}{6} + \frac{1}{2x}, \quad \frac{1}{2} \leq x \leq 1 \)

23–26 Use Simpson’s Rule with \( n = 10 \) to estimate the arc length of the curve. Compare your answer with the value of the integral produced by your calculator.

23. \( y = xe^{-x}, \quad 0 \leq x \leq 5 \)

24. \( x = y + \sqrt{y}, \quad 1 \leq y \leq 2 \)

25. \( y = \sec x, \quad 0 \leq x \leq \pi/3 \)

26. \( y = x \ln x, \quad 1 \leq x \leq 3 \)
27. (a) Graph the curve \( y = x \sqrt{4 - x} \), \( 0 \leq x \leq 4 \).
(b) Compute the lengths of inscribed polygons with \( n = 1, 2, \) and 4 sides. (Divide the interval into equal subintervals.) Illustrate by sketching these polygons (as in Figure 6).
(c) Set up an integral for the length of the curve.
(d) Use your calculator to find the length of the curve to four decimal places. Compare with the approximations in part (b).

28. Repeat Exercise 27 for the curve
\[ y = x + \sin x \quad 0 \leq x \leq 2\pi \]

29. Use either a computer algebra system or a table of integrals to find the exact length of the arc of the curve \( y = \ln x \) that lies between the points (1, 0) and (2, ln 2).

30. Use either a computer algebra system or a table of integrals to find the exact length of the arc of the curve \( y = x^{4/3} \) that lies between the points (0, 0) and (1, 1). If your CAS has trouble evaluating the integral, make a substitution that changes the integral into one that the CAS can evaluate.

31. Sketch the curve with equation \( x^{2/3} + y^{2/3} = 1 \) and use symmetry to find its length.

32. (a) Sketch the curve \( y^3 = x^2 \).
(b) Use Formulas 3 and 4 to set up two integrals for the arc length from (0, 0) to (1, 1). Observe that one of these is an improper integral and evaluate both of them.
(c) Find the length of the arc of this curve from (−1, 1) to (8, 4).

33. Find the arc length function for the curve \( y = 2x^{3/2} \) with starting point \( P_0(1, 2) \).

34. (a) Graph the curve \( y = \frac{1}{3} x^3 + 1/(4x), x > 0 \).
(b) Find the arclength function for this curve with starting point \( P_0(1, \frac{1}{3}) \).
(c) Graph the arclength function.

35. Find the arc length function for the curve
\[ y = \sin^{-1}x + \sqrt{1 - x^2} \] with starting point (0, 1).

36. A steady wind blows a kite due west. The kite’s height above ground from horizontal position \( x = 0 \) to \( x = 80 \) ft is given by \( y = 150 - \frac{1}{8}(x - 50)^2 \). Find the distance traveled by the kite.

37. A hawk flying at 15 m/s at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation
\[ y = 180 - \frac{x^2}{45} \]
until it hits the ground, where \( y \) is its height above the ground and \( x \) is the horizontal distance traveled in meters. Calculate the distance traveled by the prey from the time it is dropped until the time it hits the ground. Express your answer correct to the nearest tenth of a meter.

38. The Gateway Arch in St. Louis (see the photo on page 256) was constructed using the equation
\[ y = 211.49 - 20.96 \cosh 0.03291765x \]
for the central curve of the arch, where \( x \) and \( y \) are measured in meters and \( |x| \leq 91.20 \). Set up an integral for the length of the arch and use your calculator to estimate the length correct to the nearest meter.

39. A manufacturer of corrugated metal roofing wants to produce panels that are 28 in. wide and 2 in. thick by processing flat sheets of metal as shown in the figure. The profile of the roofing takes the shape of a sine wave. Verify that the sine curve has equation \( y = \sin(\pi x/7) \) and find the width \( w \) of a flat metal sheet that is needed to make a 28-inch panel. (Use your calculator to evaluate the integral correct to four significant digits.)

40. (a) The figure shows a telephone wire hanging between two poles at \( x = -3 \) and \( x = 3 \). It takes the shape of a catenary with equation \( y = c + a \cosh(x/a) \). Find the length of the wire.
(b) Suppose two telephone poles are 50 ft apart and the length of the wire between the poles is 51 ft. If the lowest point of the wire must be 20 ft above the ground, how high up on each pole should the wire be attached?

41. Find the length of the curve
\[ y = \int_1^4 \sqrt{1 + 1} \, dt \quad 1 \leq x \leq 4 \]

42. The curves with equations \( x^n + y^n = 1, n = 4, 6, 8, \ldots \), are called fat circles. Graph the curves with \( n = 2, 4, 6, 8 \), and 10 to see why. Set up an integral for the length \( L_{2k} \) of the fat circle with \( n = 2k \). Without attempting to evaluate this integral, state the value of \( \lim_{k \to \infty} L_{2k} \).
Exercises

1. (a) 8  
2. (b) 5.7

3. \[ \frac{7}{2} + \pi/4 \]  
5. 3

7. \( f \) is c, \( f' \) is b, \( \int_0^b f(t) \, dt \) is a

9. 37  11. \( \frac{9}{25} \)  13. -76  15. \( \frac{21}{4} \)  17. Does not exist

19. \( \frac{1}{2} \sin 1 \)  21. 0  23. \( -(1/x) - 2 \ln |x| + x + C \)

25. \( \sqrt{x^2 + 4x + C} \)  27. \( [1/(2\pi)] \sin \pi t + C \)

29. \( 2e^{-x^2} + C \)  31. \( -\frac{1}{2} \ln(\cos x)^2 + C \)

33. \( \frac{1}{2} \ln(1 + x^4) + C \)  37. \( \frac{23}{3} \)

39. \( 2\sqrt{1 + \sin x} + C \)

41. \( \frac{64}{5} \)  43. \( F'(x) = x^2/(1 + x^3) \)

45. \( \int_0^1 x^4 \, dx = 4 - e^{-x} \sqrt{2} \)

57. Number of barrels of oil consumed from Jan. 1, 2000, through Jan. 1, 2008

59. 72,400  61. 3  63. \( c = 1.62 \)

65. \( f(x) = e^{2/(1 + 2x)}/(1 - e^{-x}) \)

71. \( \frac{7}{2} \)

PROBLEMS PLUS  PAGE 413

1. \( \pi/2 \)  3. \( f(x) = \frac{1}{2}x \)  5. -1  7. \( e^{-2} \)

11. (a) \( \frac{1}{2}(n - 1)n \)  (b) \( \frac{3}{2}[2b - [b] - 1] - \frac{3}{2}a(2a - [a] - 1) \)

17. \( 2(\sqrt{2} - 1) \)

CHAPTER 6

EXERCISES 6.1  PAGE 420

1. \( \frac{12}{7} \)  3. \( e - (1/e) + \frac{10}{3} \)  5. 19.5  7. \( \frac{1}{6} \)  9. \( \ln 2 - \frac{1}{2} \)

11. \( \frac{2}{3} \)  13. 72  15. 2 - 2 \ln 2  17. \( \frac{29}{12} \)  19. \( \frac{32}{7} \)

21. \( \frac{8}{7} \)  23. \( \frac{1}{2} \)  25. \( \pi - \frac{2}{7} \)  27. \( \ln 2 \)  29. 6.5

31. \( \frac{1}{2}\sqrt{3} - 1 \)  33. 0.6407  35. 0, 0.90; 0.04  37. 8.38

39. \( 12\sqrt{6} - 9 \)  41. 117.4 ft  43. 4232 cm²

45. (a) Car A  (b) The distance by which A is ahead of B after 1 minute  (c) Car A  (d) \( t \approx 2.2 \) min

47. \( \frac{1}{2}\sqrt{3} \)  49. \( 4\sqrt[3]{3} \)  51. \( \pm 6 \)

53. \( 0 < m < 1; m - \ln m - 1 \)

EXERCISES 6.2  PAGE 430

1. \( 19\pi/12 \)

13. \( 2\pi(\frac{1}{3} \pi - \sqrt{3}) \)
15. 16\pi/15

17. 29\pi/30

19. \pi/7
21. \pi/10
23. \pi/2
25. 7\pi/15
27. 5\pi/14
29. 13\pi/30
31. \pi \int_0^{\pi/4} (1 - \tan x)^2 \, dx
33. \pi \int_0^\pi (1^2 - (1 - \sin x)^2) \, dx
35. \pi \int_{-2}^2 \left[ y^2 - (\sqrt{1 + y^2} + 2)^2 \right] \, dy
37. -1.288, 0.884; 23.780
41. Solid obtained by rotating the region 0 ≤ y ≤ \cos x, 0 ≤ x ≤ \pi/2 about the x-axis
43. Solid obtained by rotating the region above the x-axis bounded by x = y^2 and x = y^4 about the y-axis
45. 1110 cm^3

EXERCISES 6.3 = PAGE 436
1. Circumference = 2\pi x, height = x(x - 1)^2; \pi/15

3. 2\pi

5. \pi(1 - 1/e)

7. 16\pi

9. 21\pi/2

EXERCISES 6.4 = PAGE 441
1. 588 J
3. 9 ft-lb
5. 180 J
7. 15 \frac{3}{4} ft-lb
9. (a) \frac{25}{34} \approx 1.04 J
(b) 10.8 cm
11. W_2 = 3W_1
13. (a) 625 ft-lb
(b) \frac{825}{3} ft-lb
15. 650,000 ft-lb
17. 3857 J
19. 2450 J
21. \approx 1.06 \times 10^6 J
23. \approx 1.04 \times 10^5 ft-lb
25. 2.0 m
29. Gm_1m_2 \left( \frac{1}{a} - \frac{1}{b} \right)
57. \( \frac{1}{2} \sin x \sqrt{4 + \sin^2 x} + 2 \ln(\sin x + \sqrt{4 + \sin^2 x}) + C \)
61. No
63. (a) 1.925444 (b) 1.920915 (c) 1.922470
65. (a) 0.01348, \( n \geq 368 \)  (b) 0.00674, \( n \geq 260 \)
67. 8.6 mi
69. (a) 3.8  (b) 1.7867, 0.000646  (c) \( n \approx 30 \)
71. C  73. 2  75. \( \frac{5}{10} \pi^2 \)

**PROBLEMS PLUS**  **PAGE 521**

1. About 1.85 inches from the center  3. \( 0 \)
7. \( f(\pi) = -\pi/2 \)  11. \( (b^2 a^{-3})^{1/0.001} e^{-1} \)
13. \( 2 - \sin^{-1}(2/\sqrt{5}) \)

**CHAPTER 8**

**EXERCISES 8.1  ** **PAGE 530**

1. \( 4\sqrt{5} \)  3. \( \int_0^\pi \sqrt{1 + \sin^2 x} \, dx \)  5. \( \int_0^\pi \sqrt{9y^4 + 6y^2 + 2} \, dy \)
7. \( \frac{\pi}{10} (82\sqrt{82} - 1) \)  9. \( \frac{\ln 3}{\pi} \)  11. \( \frac{25}{3} \)
13. \( \ln(\sqrt{2} + 1) \)  15. \( \ln 3 - \frac{1}{3} \)
17. \( \sqrt{1 + \varepsilon^2 - \sqrt{2 + \ln(\sqrt{1 + \varepsilon^2 - 1}) - 1 - \ln(\sqrt{2} - 1)} \)}
19. \( \frac{\sqrt{\pi}}{\pi} \)  21. \( \frac{4}{\pi} \)  23. \( 5.115840 \)
25. \( 1.569619 \)
27. (a), (b) \( 3 \)

(c) \( \int_0^\pi \sqrt{1 + (4(3 - x)/(3(4 - x)^2))} \, dx \)  (d) 7.7988
29. \( \sqrt{5} - \ln\left(\frac{1}{2}(1 + \sqrt{3})\right) - \sqrt{2} + \ln(1 + \sqrt{2}) \)
31. \( 6 \)

33. \( s(x) = \frac{2}{\pi}\left[1 + (9x)^{3/2} - 10 \sqrt{10}\right] \)
35. \( 2\sqrt{2}(\sqrt{1 + x} - 1) \)
37. 209.1 m  39. 29.36 in.  41. 12.4

**EXERCISES 8.2  ** **PAGE 537**

1. (a) \( \int_0^\pi 2\pi x^4 \sqrt{1 + 16x^2} \, dx \)  (b) \( \int_0^\pi 2\pi x \sqrt{1 + 16x^6} \, dx \)
3. (a) \( \int_0^\pi \frac{\pi}{2} \tan^{-1}x \sqrt{1 + \frac{1}{(1 + x^2)^2}} \, dx \)
(b) \( \int_0^\pi \frac{\pi}{2} \frac{1}{(1 + x^2)^2} \, dx \)
5. \( \frac{1}{\pi} \pi (145 \sqrt{145} - 1) \)  7. \( \frac{1}{10} \pi^2 \)

9. \( 2\sqrt{1 + \pi^2} + (2/\pi) \ln(\pi + \sqrt{1 + \pi^2}) \)  11. \( \frac{2}{\pi} \)
13. \( \frac{\pi}{3} (145 \sqrt{145} - 10 \sqrt{10}) \)  15. \( \pi a^2 \)
17. 9.023754  19. 13.52796
21. \( \frac{1}{\pi} \left[4 \ln(\sqrt{17} + 4) - 4 \ln(\sqrt{2} + 1) - \sqrt{17} + 4\sqrt{2}\right] \)
23. \( \frac{1}{\pi} \left[\ln(\sqrt{10} + 3) + 3 \sqrt{10}\right] \)
27. (a) \( \frac{1}{\pi} a^2 \)  (b) \( \frac{\pi}{\sqrt{\pi}} \pi^2 \)
29. (a) \( 2\pi \left[b^2 + a^2 b \sin^{-1} \left(\sqrt{a^2 - b^2}/a\right)\right] \)
(b) \( 2\pi \left[a^2 + a b \sin^{-1} \left(\sqrt{b^2 - a^2}/b\right)\right] \)
31. \( \int_0^\pi 2\pi e^{-f(x)} \sqrt{1 + [f'(x)]^2} \, dx \)  33. \( 4\pi^2 r^2 \)

**EXERCISES 8.3  ** **PAGE 547**

1. (a) 187.5 lb/ft²  (b) 1875 lb  (c) 562.5 lb
5. 6000 lb  5. \( 6.7 \times 10^4 \) N  7. \( 9.8 \times 10^4 \) N
9. \( 1.2 \times 10^4 \) lb  11. \( \frac{3}{4} \)  13. \( 5.27 \times 10^5 \) N
15. (a) 314 N  (b) 353 N
17. (a) 5.63 \( \times 10^3 \) lb  (b) 5.06 \( \times 10^4 \) lb
(c) \( 4.88 \times 10^4 \) lb  (d) \( 3.03 \times 10^3 \) lb
19. \( 2.5 \times 10^5 \) N  21. 230; \( \frac{3}{2} \)  23. 10; \( 1; \frac{1}{4}; \frac{10}{37} \)
25. (0, 1.6)  27. \( \left(\frac{1}{e - 1}, \frac{e + 1}{4}\right) \)  29. \( \left(\frac{m}{\pi}, \frac{m}{\pi}\right) \)
31. \( \left(\frac{\pi \sqrt{2} - 4}{4(\sqrt{2} - 1)}, \frac{1}{4(\sqrt{2} - 1)}\right) \)  33. (2, 0)
35. 60; 160; (\( \frac{1}{2}, 1 \))  37. (0.781, 1.330)  41. (0, \( \frac{1}{2} \))
45. \( \frac{1}{10} \pi^2 \)

**EXERCISES 8.4  ** **PAGE 553**

1. \( \$38,000 \)  3. \( \$43,866,933.33 \)  5. \( \$407.25 \)
7. \( \$12,000 \)  9. 3727; \( \$37,753 \)
11. \( \frac{3}{4}(16\sqrt{2} - 8) \approx \$9.75 \) million
13. \( \frac{(1 - k)(b^{2 - k} - a^{2 - k})}{(2 - k)(b^{1 - k} - a^{1 - k})} \)
15. \( 1.19 \times 10^{-4} \) cm³/s
17. 6.60 L/min  19. 5.77 L/min

**EXERCISES 8.5  ** **PAGE 560**

1. (a) The probability that a randomly chosen tire will have a lifetime between 30,000 and 40,000 miles
(b) The probability that a randomly chosen tire will have a lifetime of at least 25,000 miles
3. (a) \( f(x) \geq 0 \) for all \( x \) and \( \int_0^\pi f(x) \, dx = 1 \)
(b) \( 1. \frac{1}{\sqrt{3}} = 0.35 \)
5. (a) \( 1/\pi \)  (b) \( \frac{1}{2} \)
7. (a) \( f(x) \geq 0 \) for all \( x \) and \( \int_0^\pi f(x) \, dx = 1 \)  (b) \( 5 \)
11. (a) \( e^{-4/25} \approx 0.20 \)  (b) \( 1 - e^{-2/25} \approx 0.55 \)  (c) If you aren’t served within 10 minutes, you get a free hamburger.
13. \( \approx 44\% \)
15. (a) 0.0668  (b) \( \approx 5.21\% \)
17. \( \approx 0.9545 \)